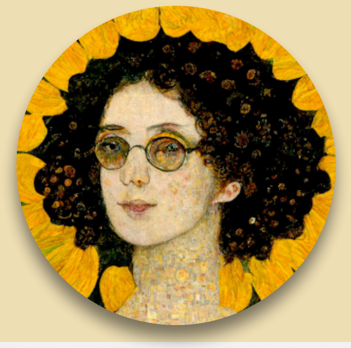
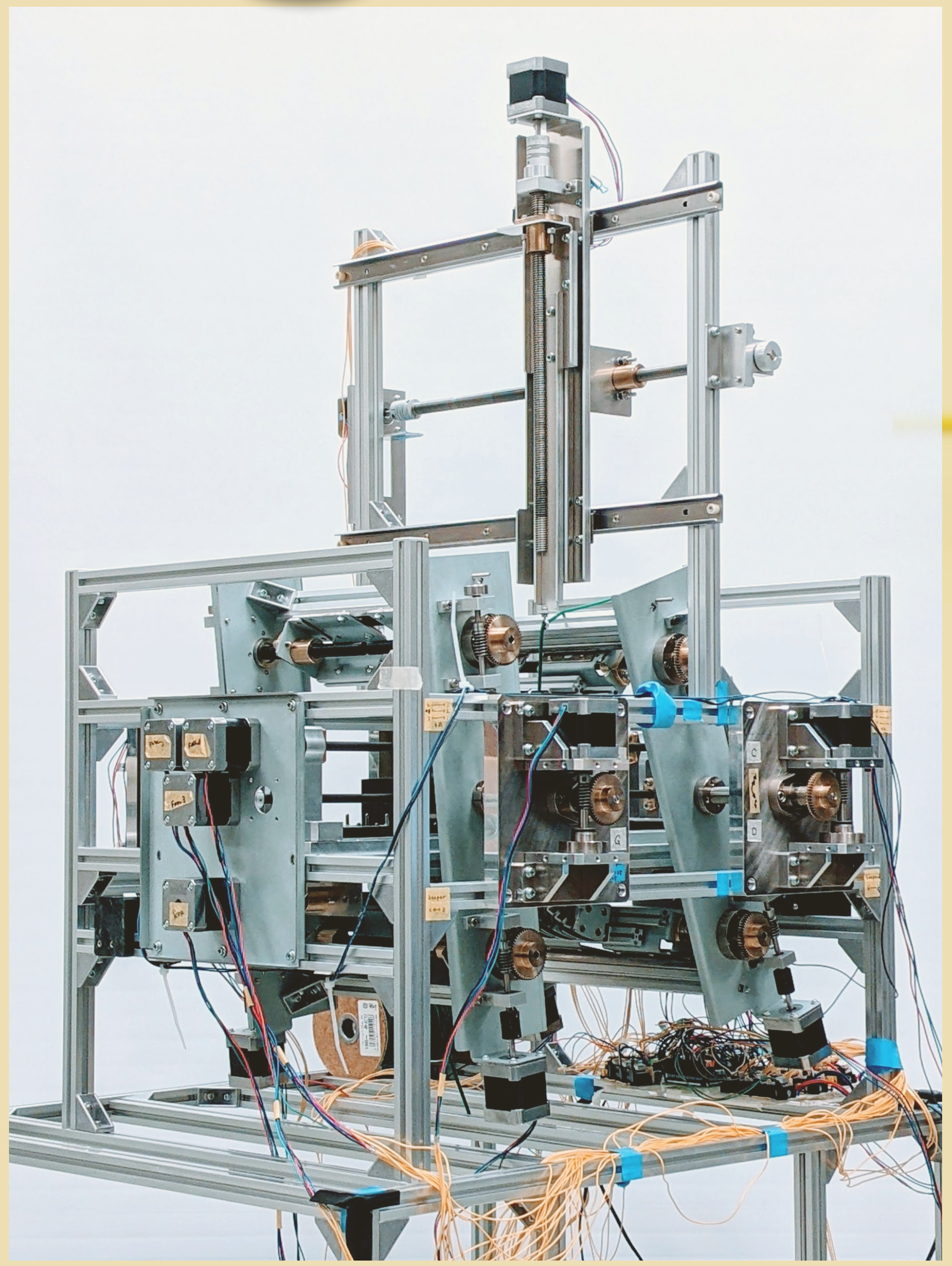
Solid Knitting

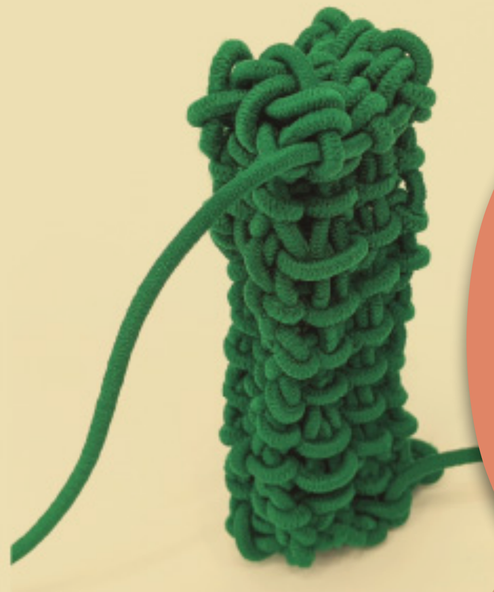

Yuichi Hirose


Mark Gillespie


Angelica Bonilla Fominaya


James McCann





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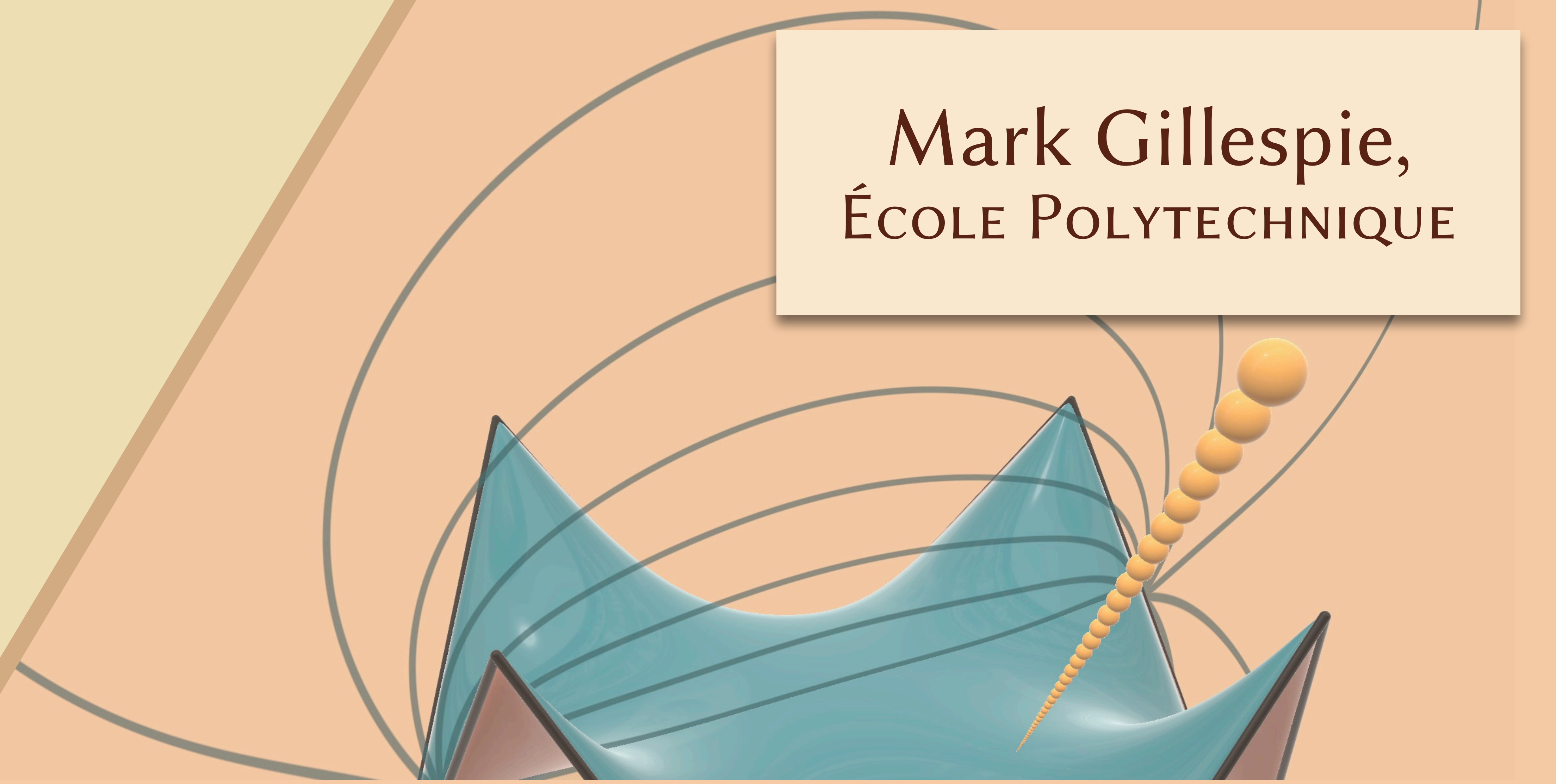


Mark Gillespie,
École Polytechnique


Mark Gillespie


Denise Yang


Mario Botsch


Keenan Crane

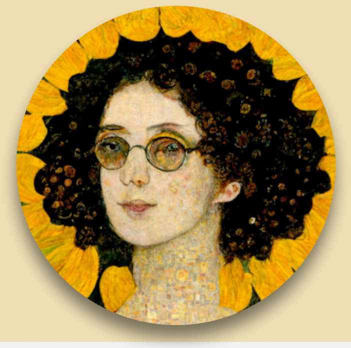
Harmonic Hitting

Mark Gillespie,
École Polytechnique

Solid Knitting

Yuichi
Hirose

Mark
Gillespie



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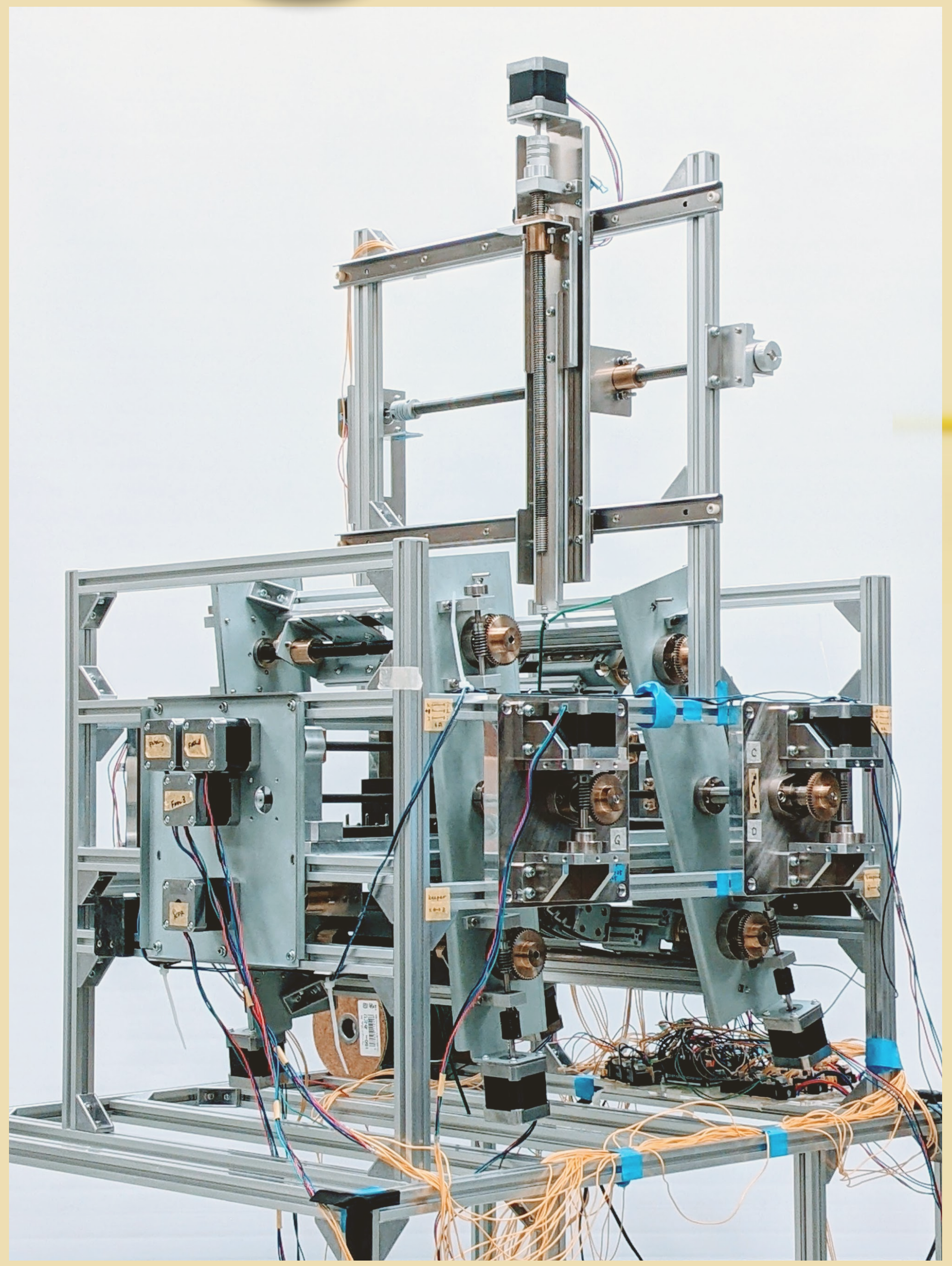

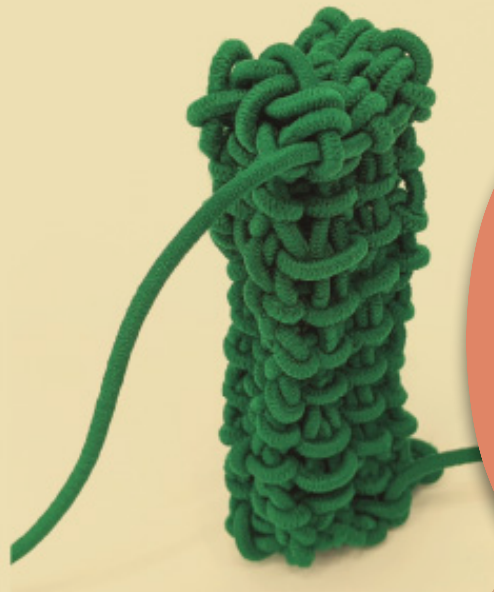

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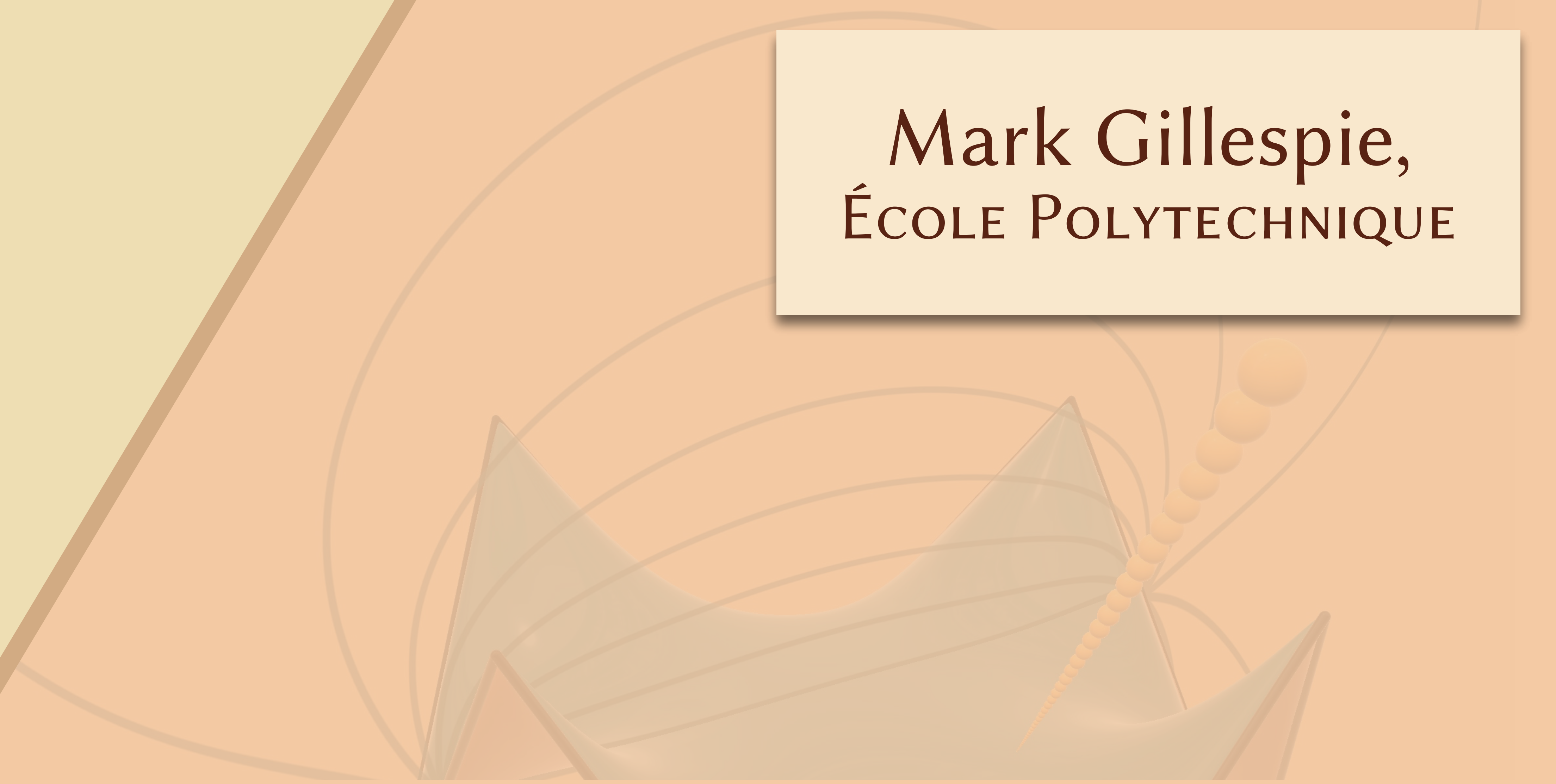

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Crane

Harmonic Hitting

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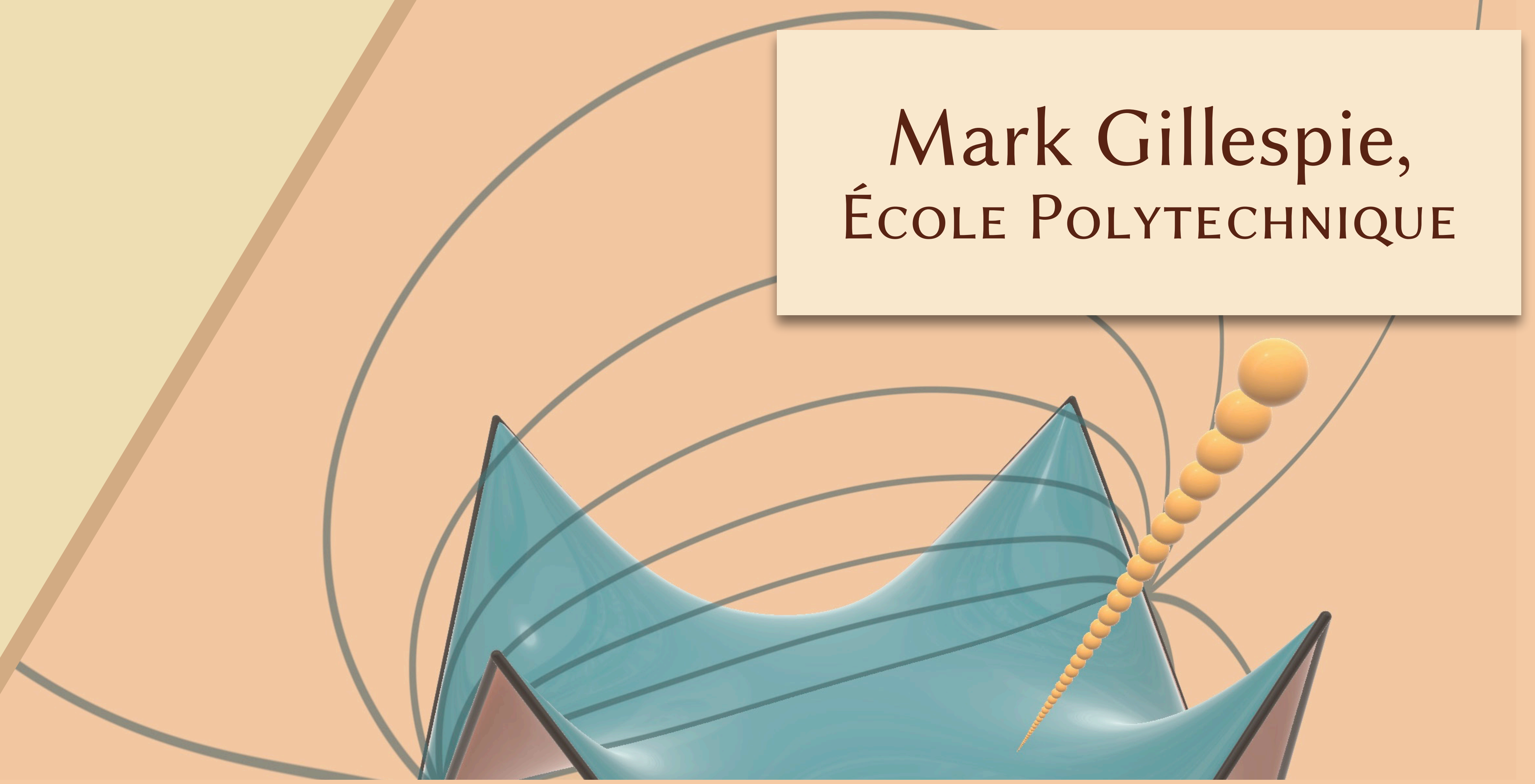

Solid Knitting


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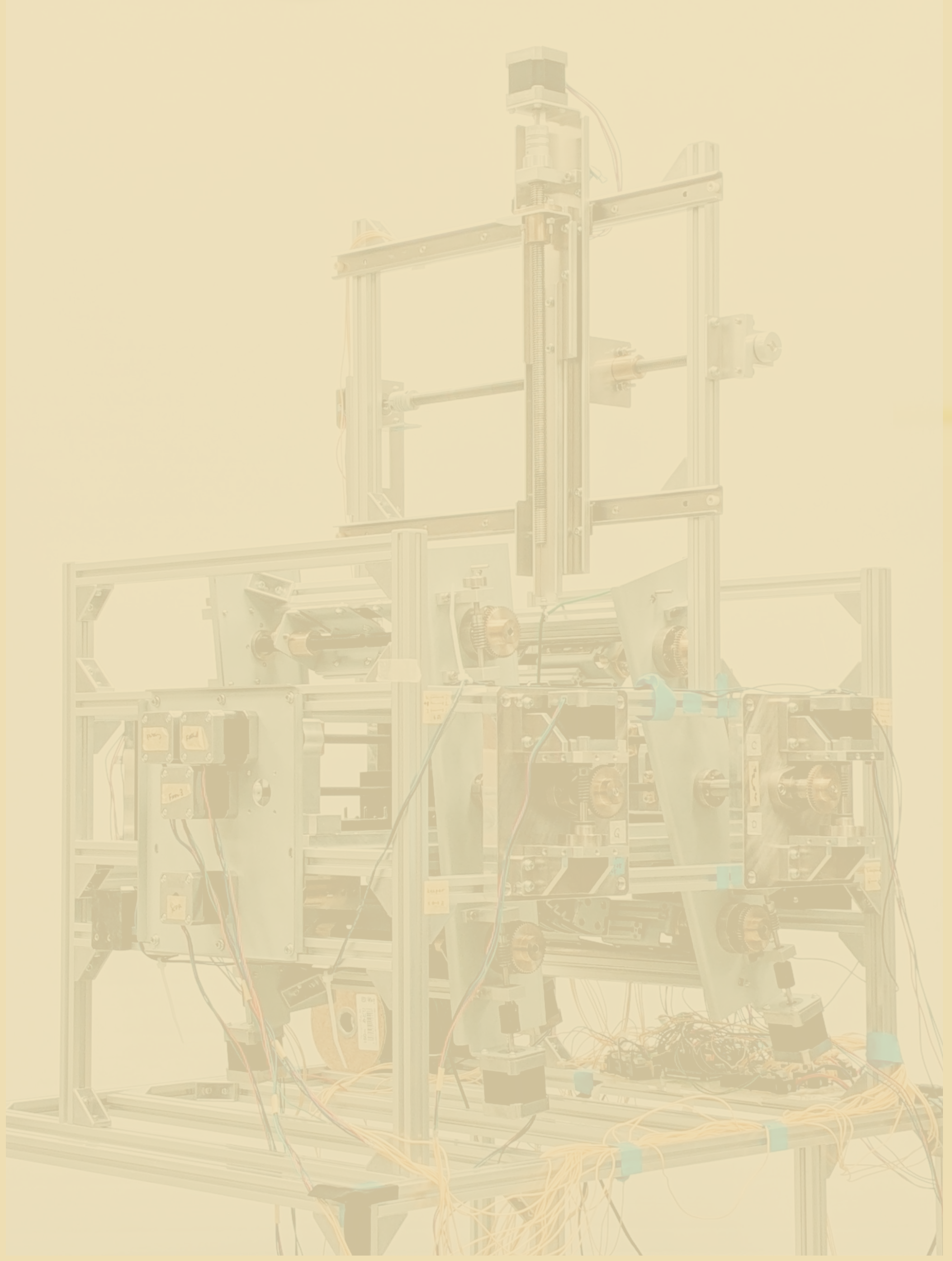

&


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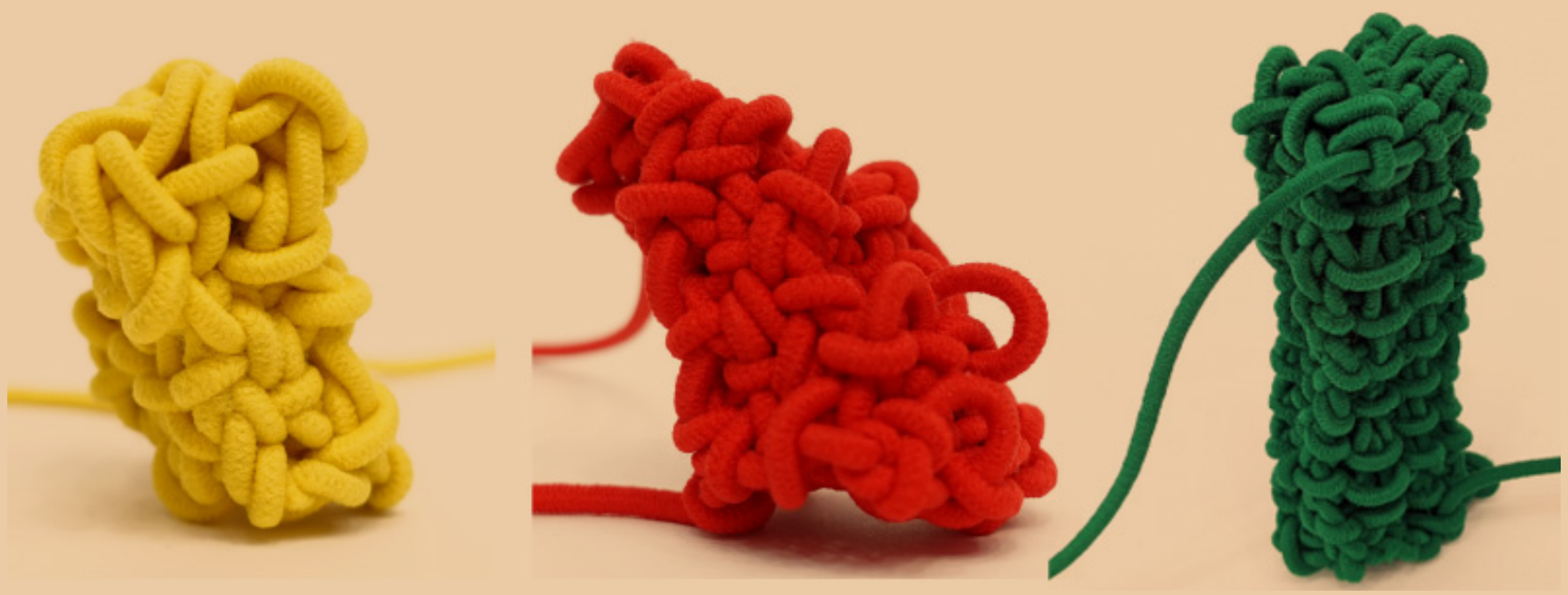
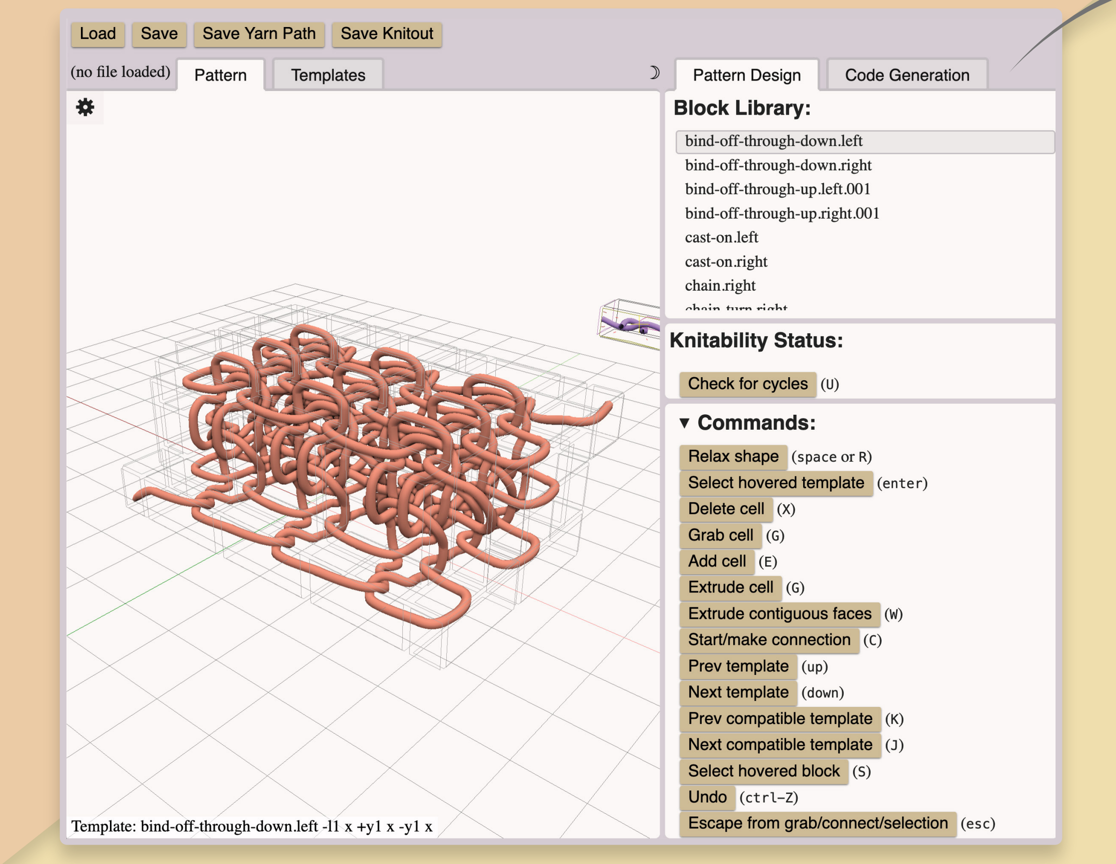
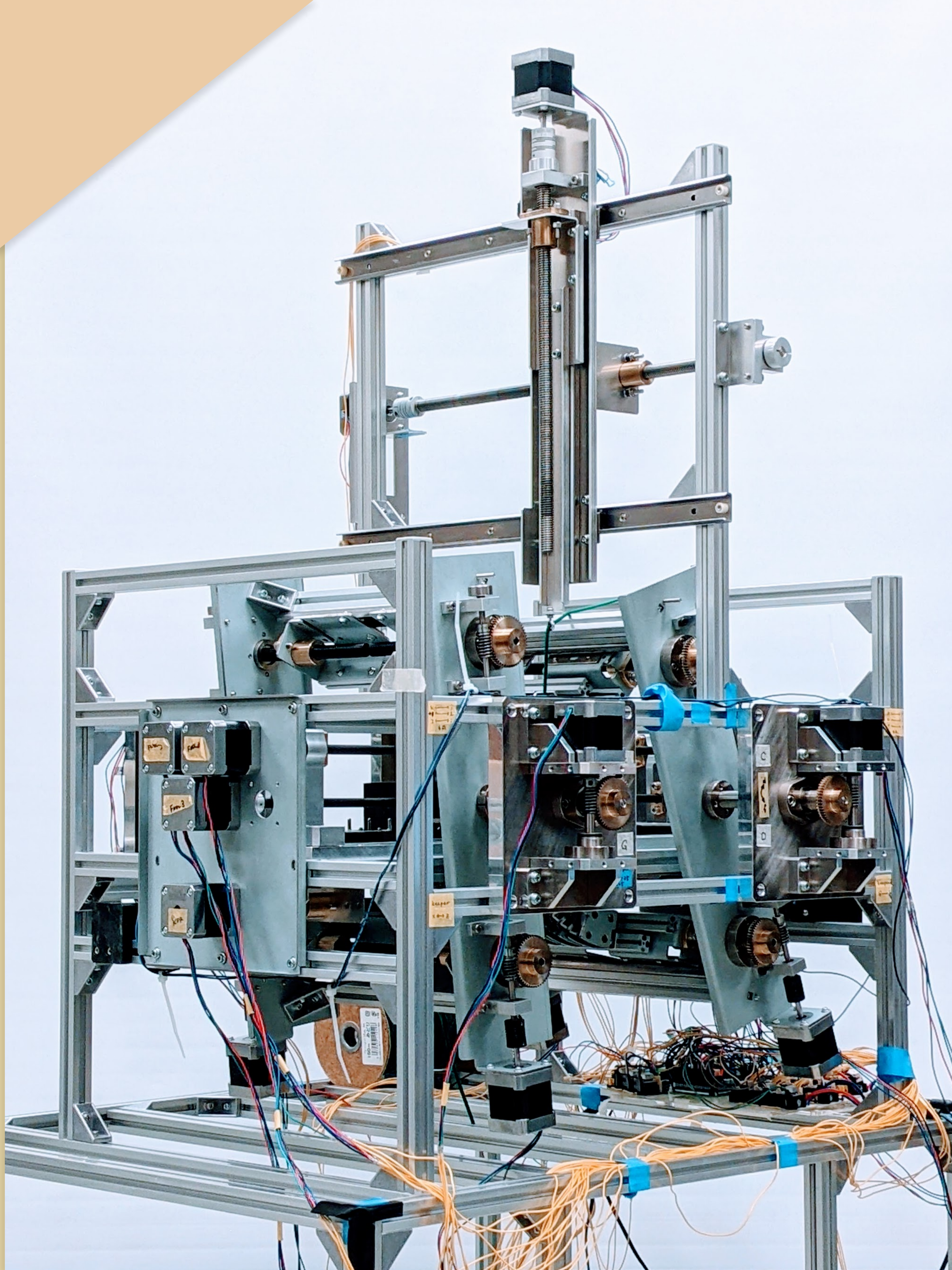
Harmonic Hitting



Solid Knitting

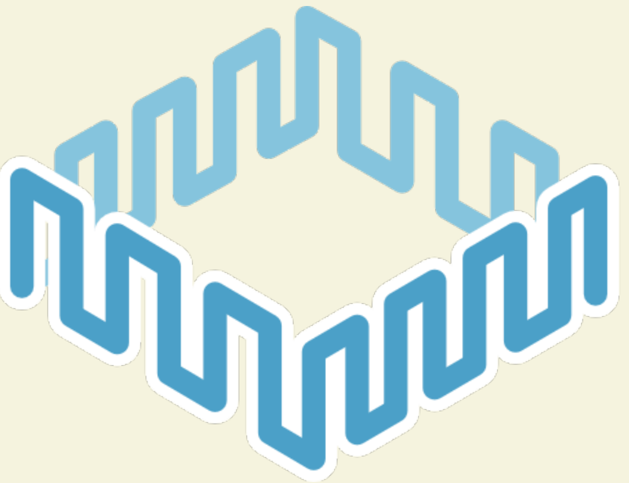


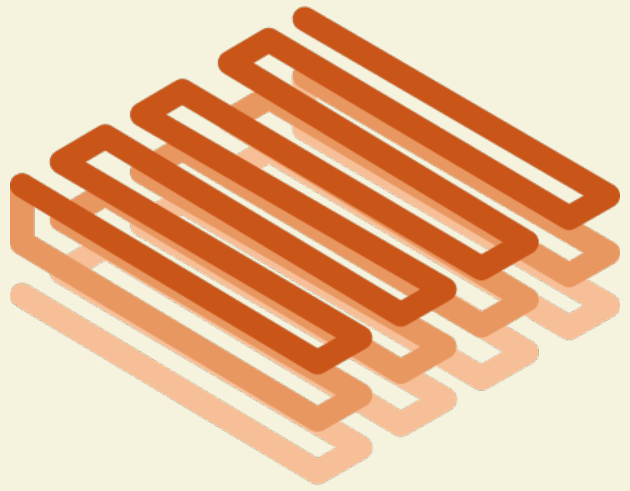

```
knit b3 hf3,6
knit b2 hf2,6
knit b1 hf1,6

xfer b3 hf3,7
xfer b2 hf2,7
xfer b1 hf1,7

xfer hf3,7 f3
xfer hf2,7 f2
xfer hf1,7 f1

release f

; ==================
;      Layer 3
; ==================
; row 0

tuck + hb1,5 1
tuck + hb2,5 1
```



*Constructed by hand

Solid knitting creates dense volumes

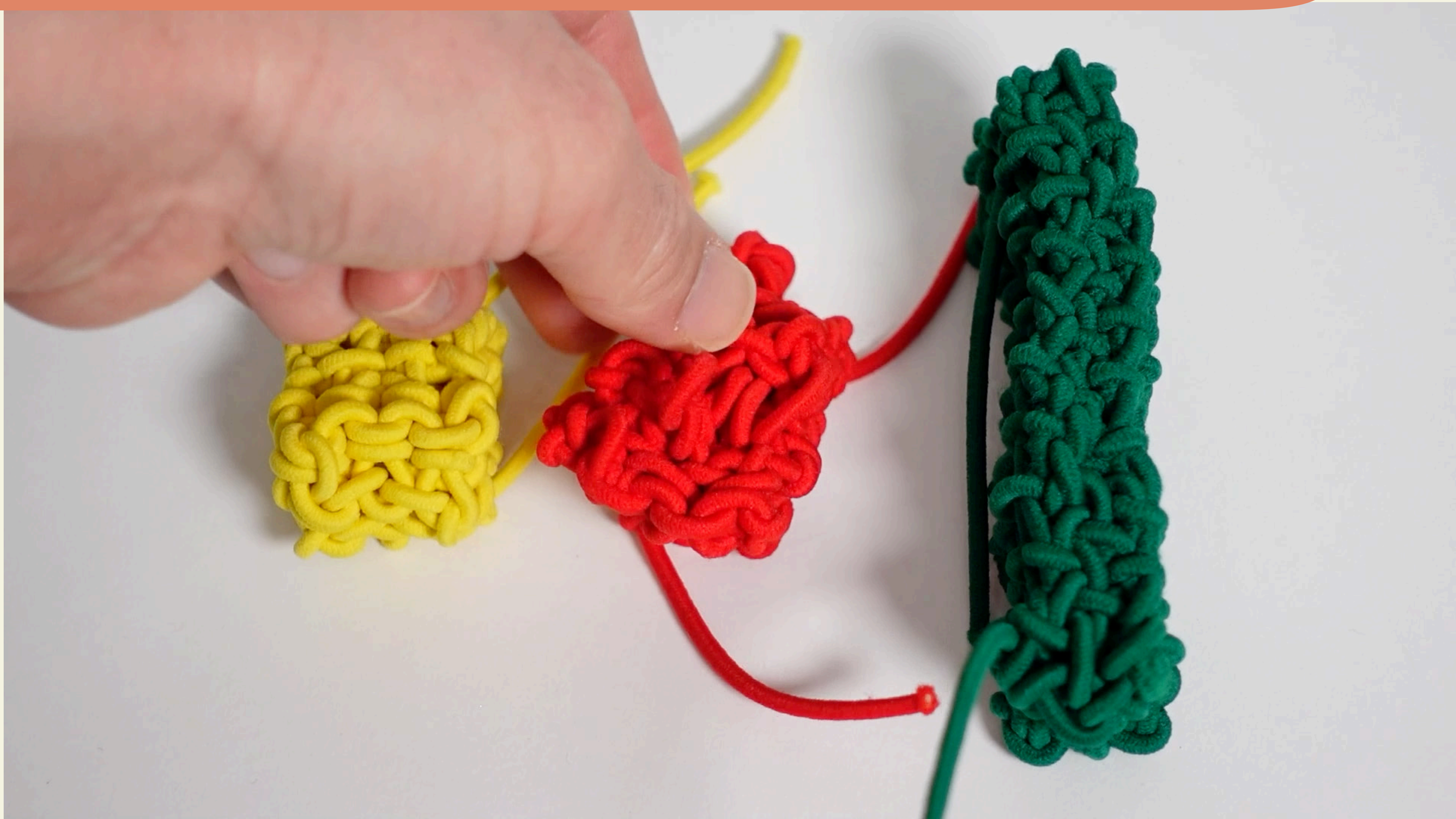




Traditional knitting:
hollow surface

Solid knitting:
dense volume

Solid knitting creates dense volumes



Solid knitting is easy to recycle

*Constructed by hand

Traditional knitting

pulling loops through loops

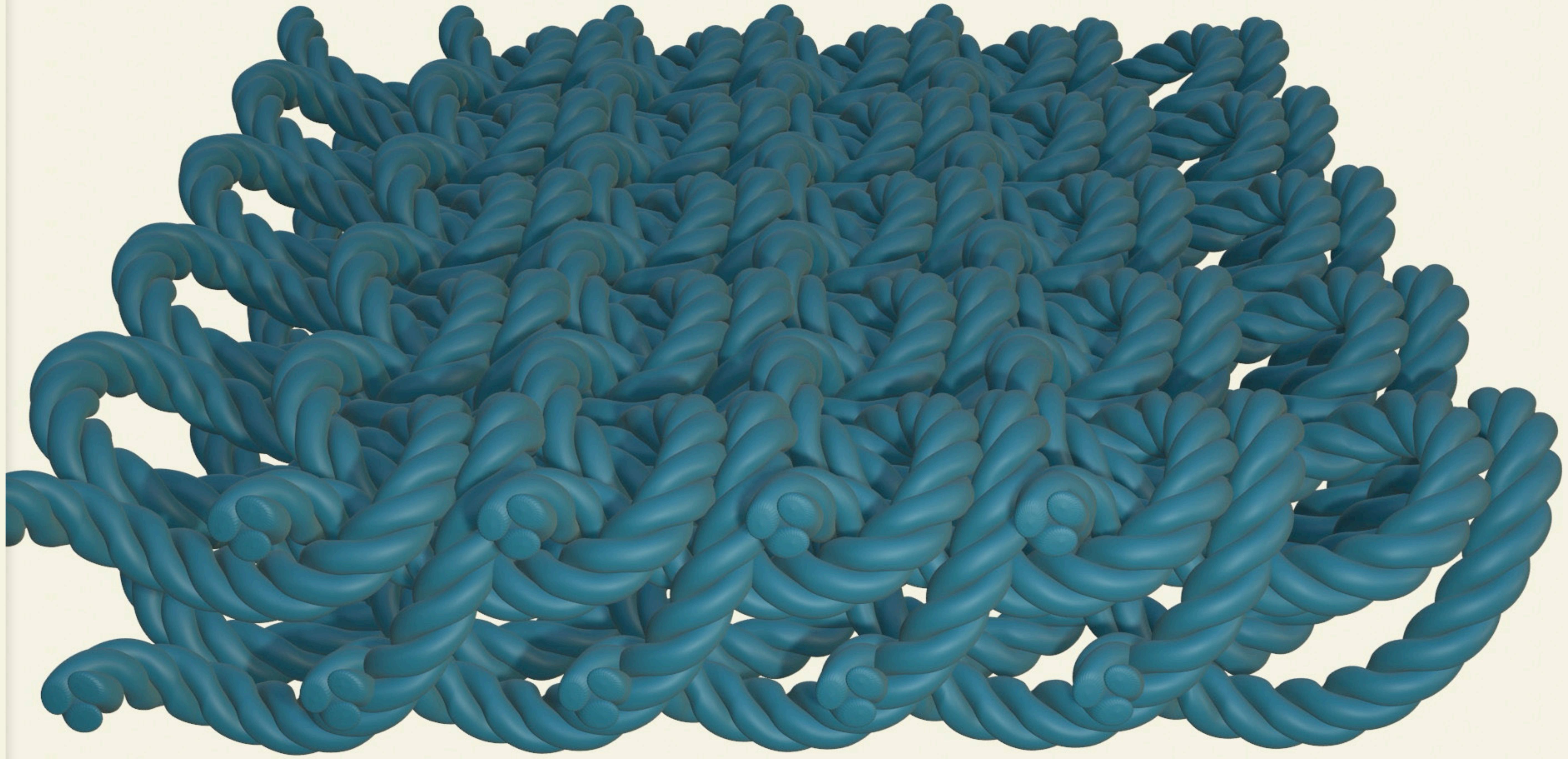
[Narayanan et al. 2018]

Solid knitting

stacking knit layers

Solid knitting

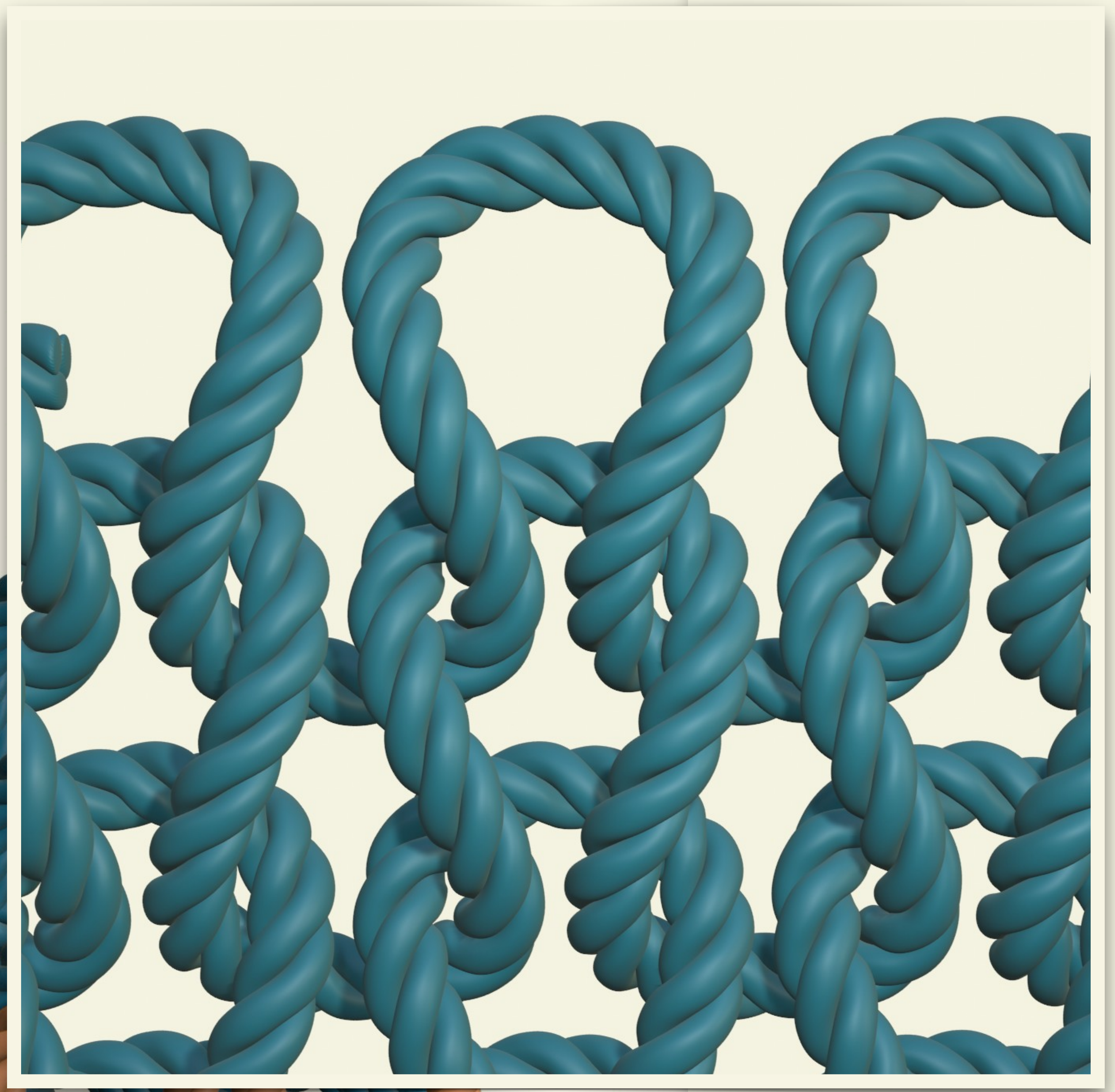
stacking knit layers



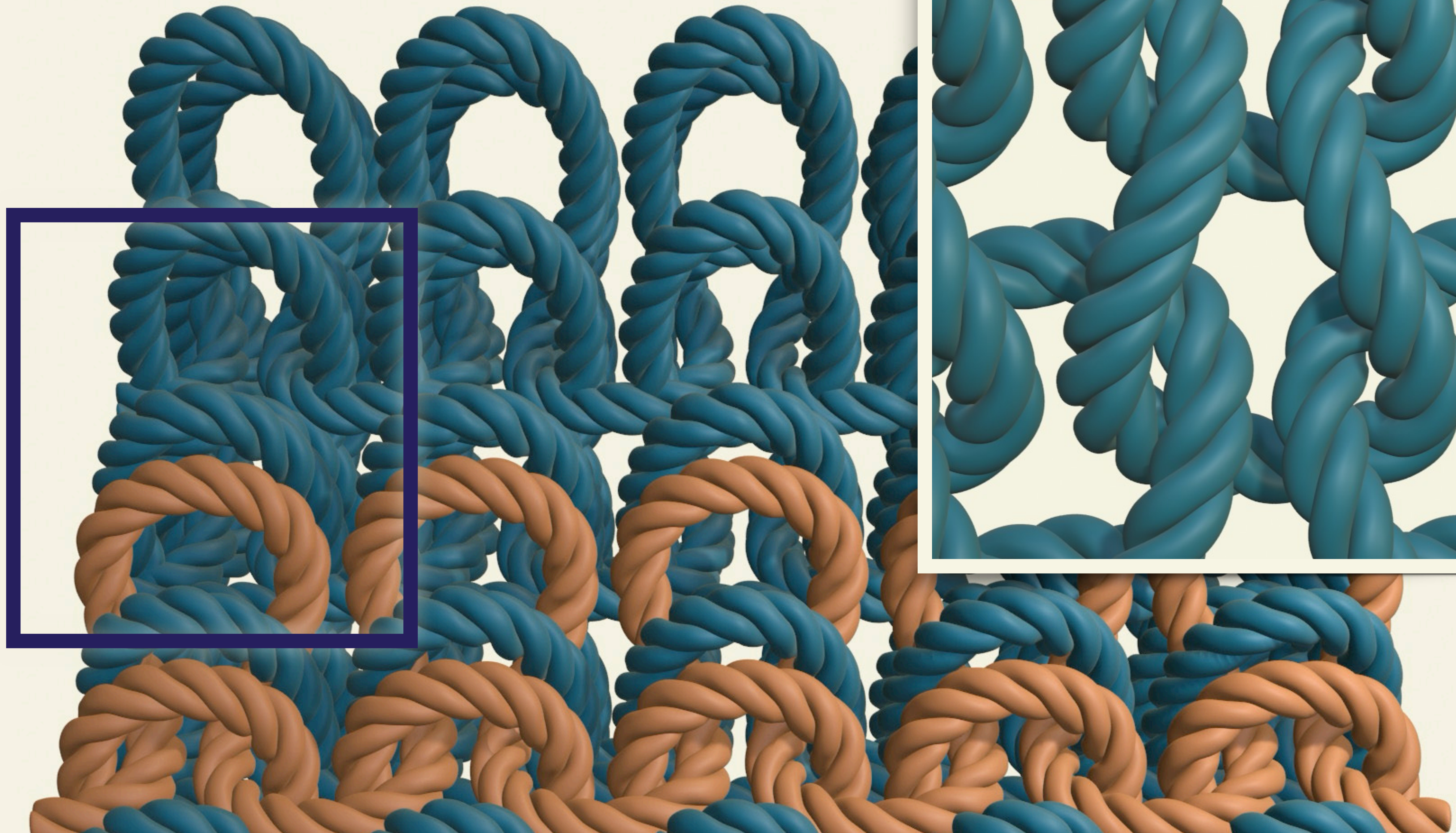
Solid knitting

stacking knit layers

traditional knitting:
pulled through *one* existing loop



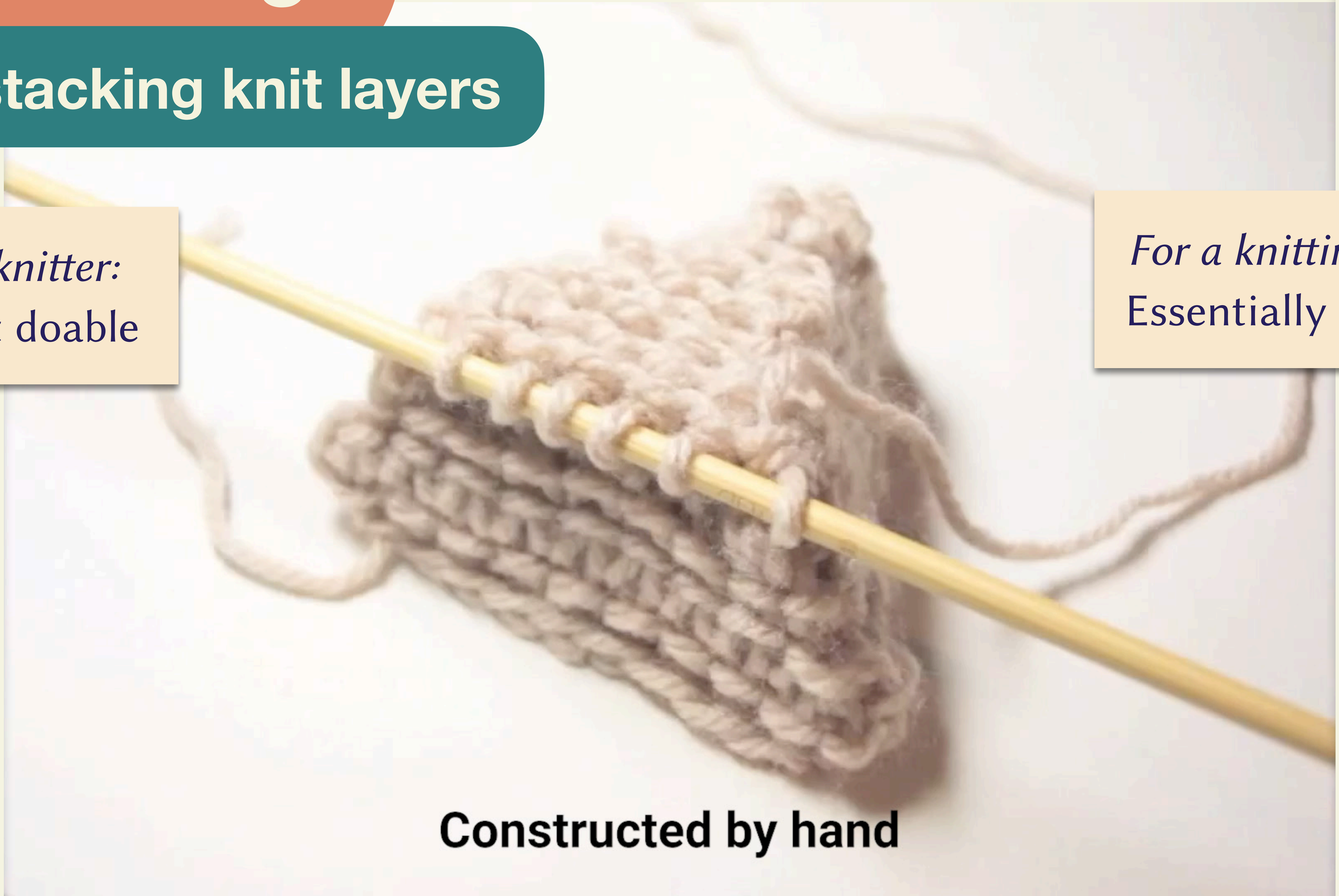
pulled through
two existing
loops



Solid knitting

stacking knit layers

For a hand knitter:
Unusual, but doable

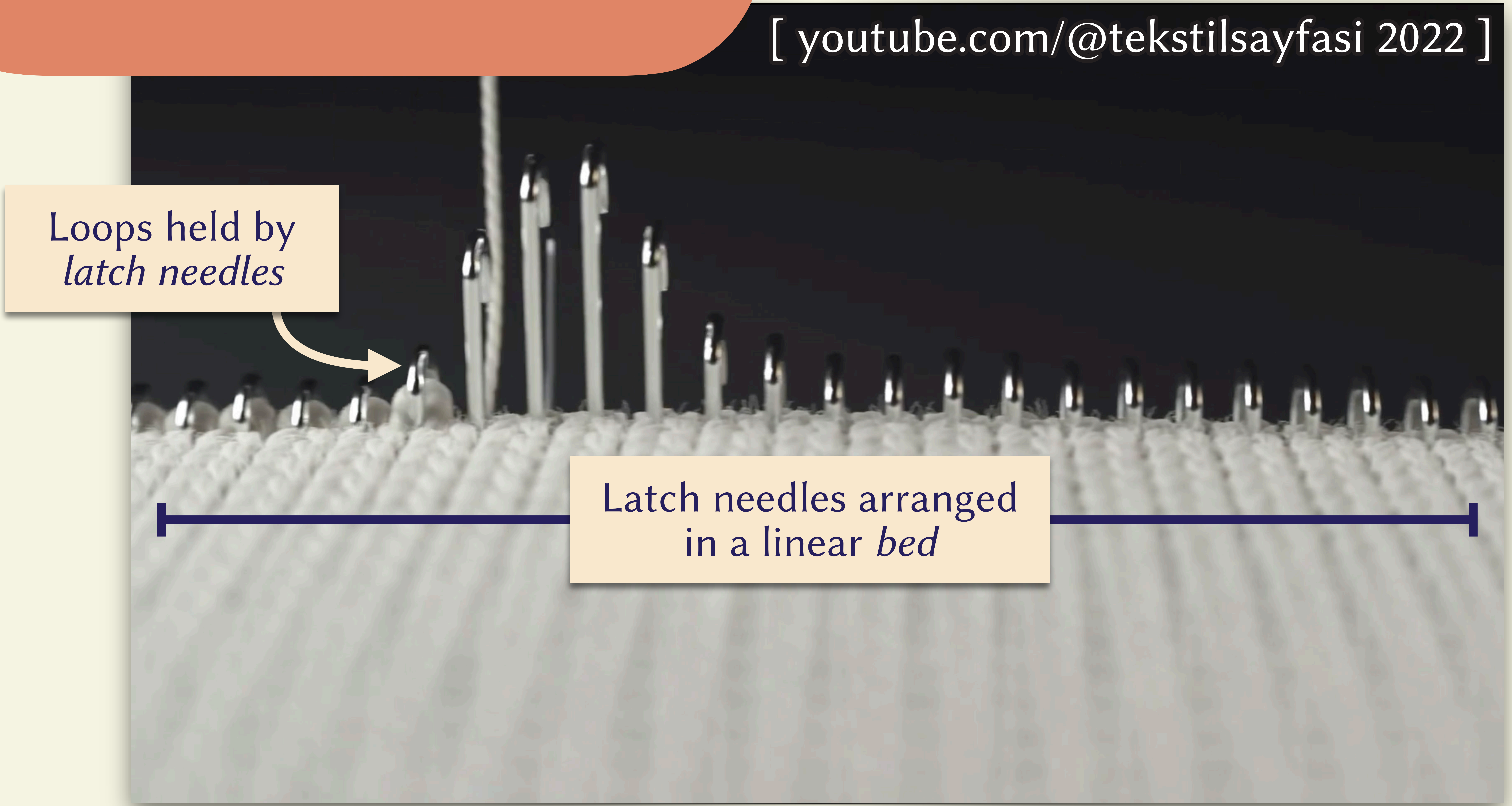


For a knitting machine:
Essentially impossible!

Knitting machines

[youtube.com/@tekstilsayfasi 2022]

Knitting machines



Challenge: storing loops

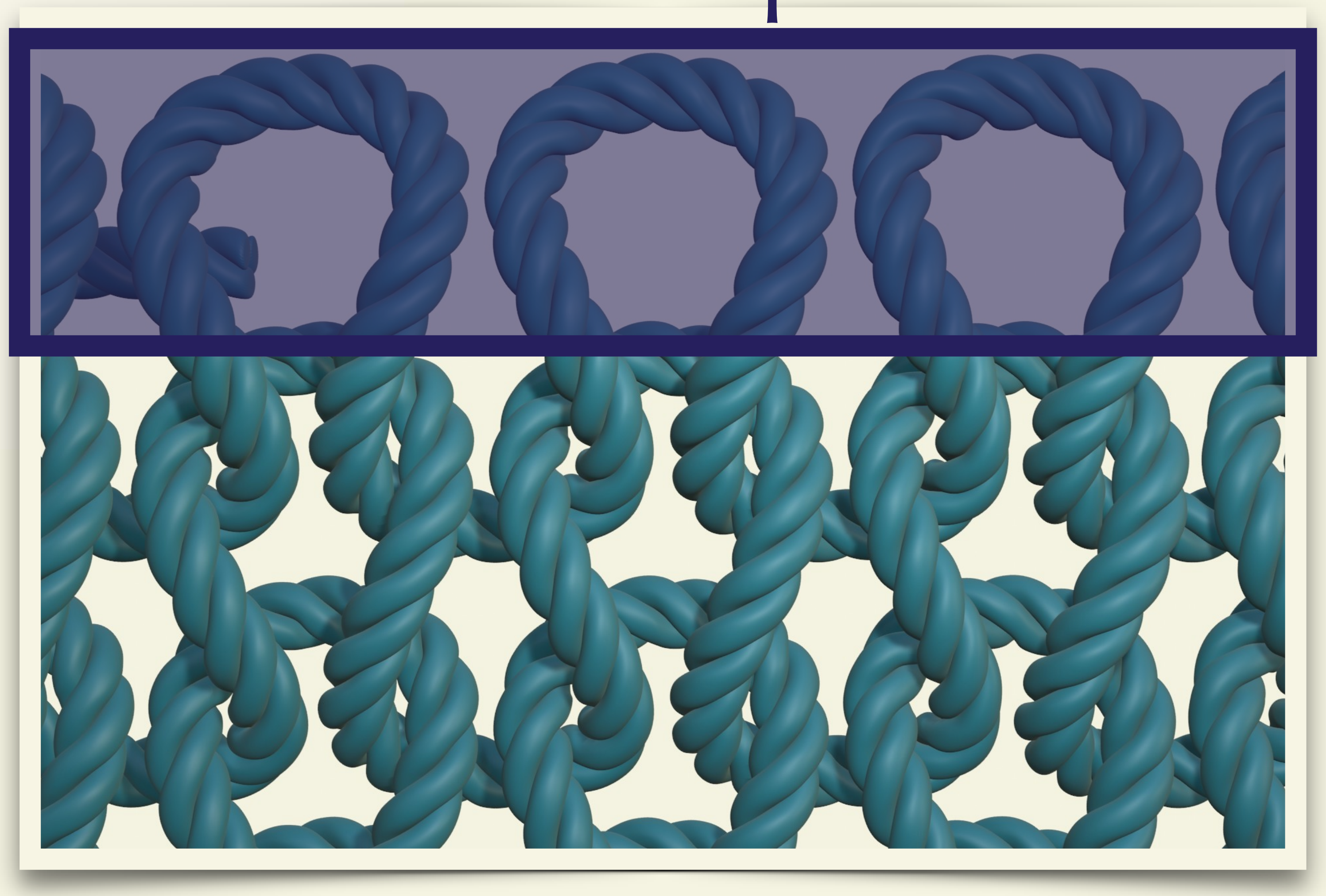
traditional knitting machines struggle to even *hold* all of these loops!

Latch needles cannot pick up loops

If we ever want to manipulate a loop in the future ...

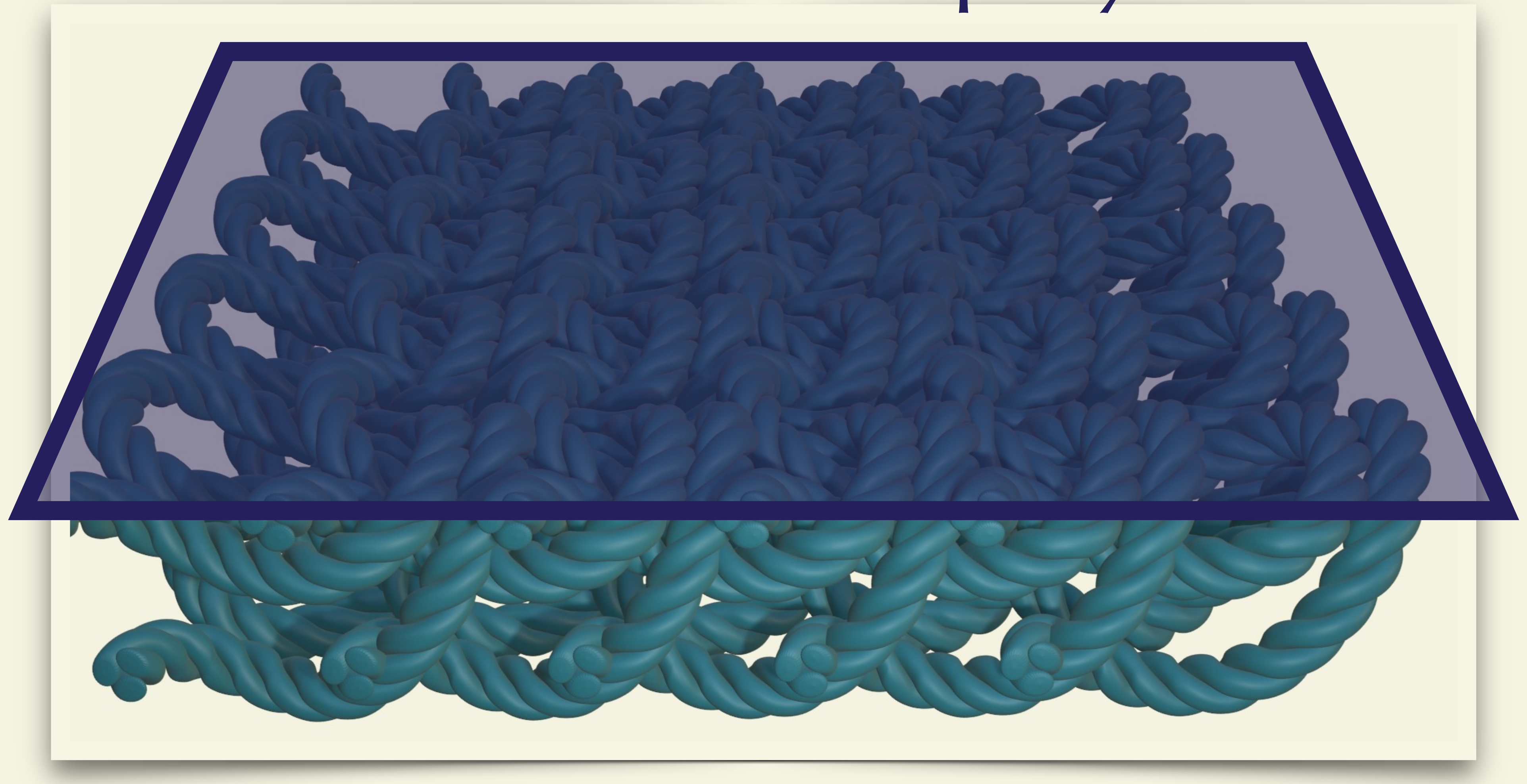
we have to hold it on a needle

hold onto top *row*



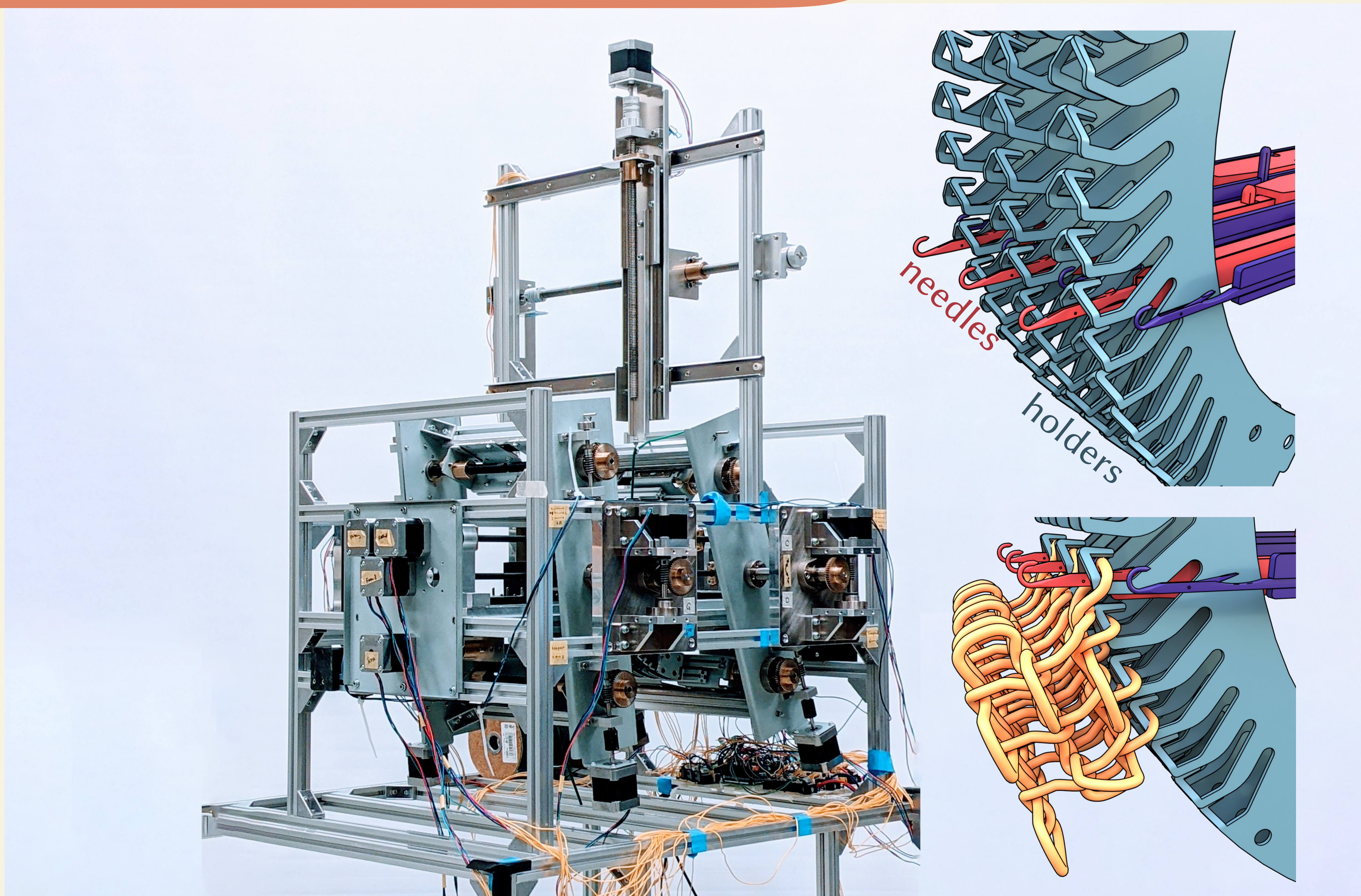
traditional knitting

hold onto entire top *layer*

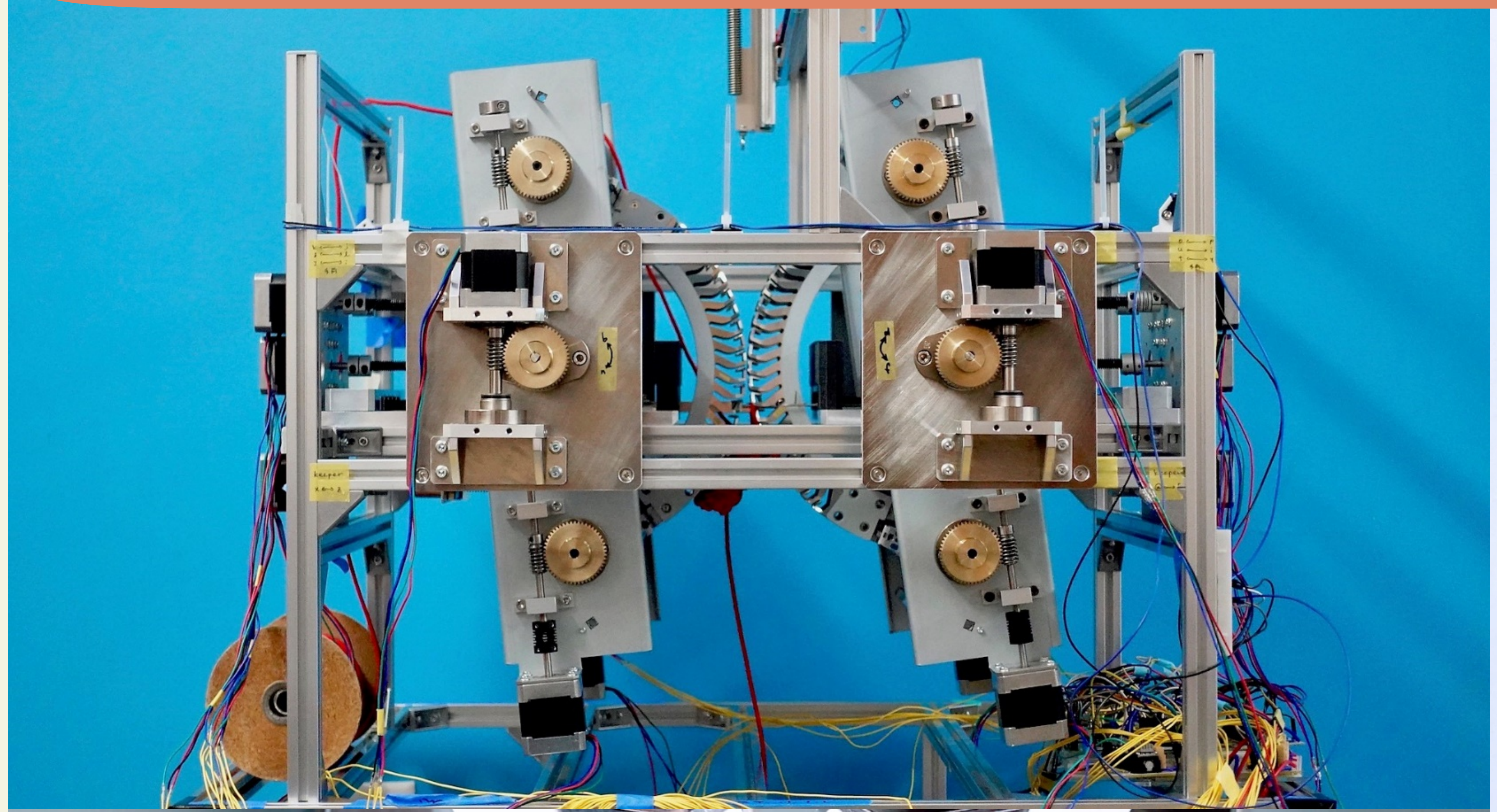


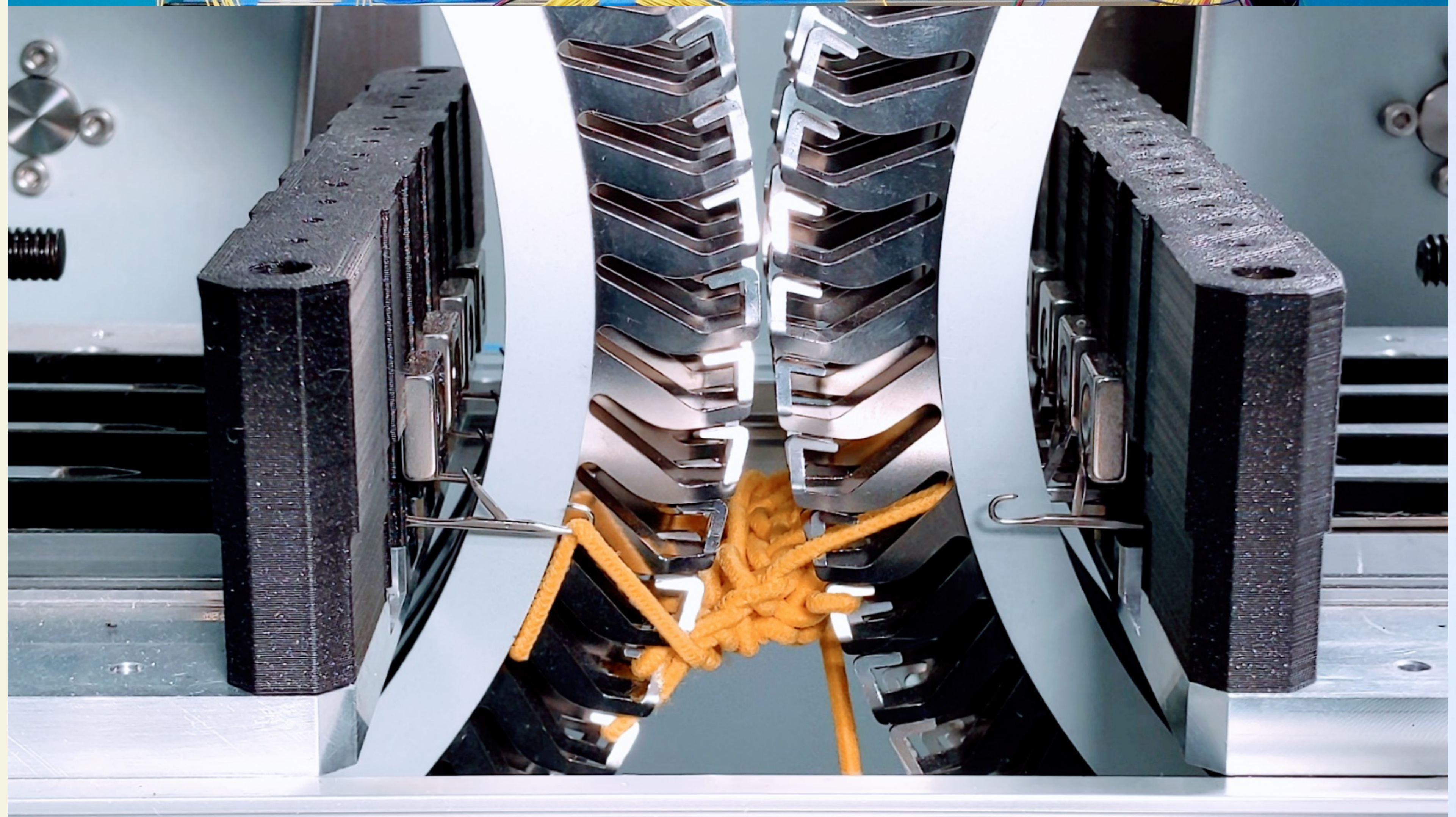
solid knitting

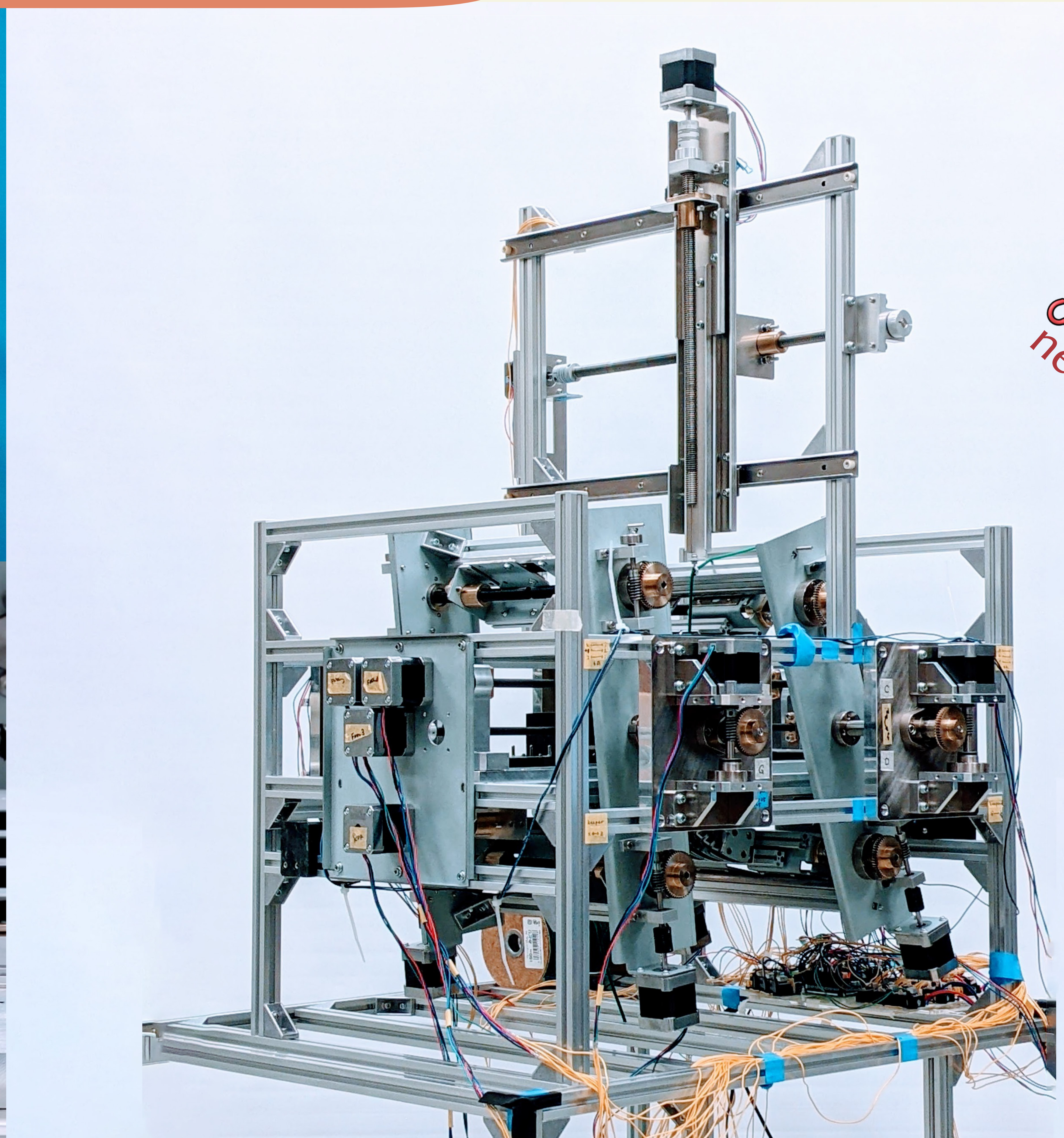
Our solid knitting machine

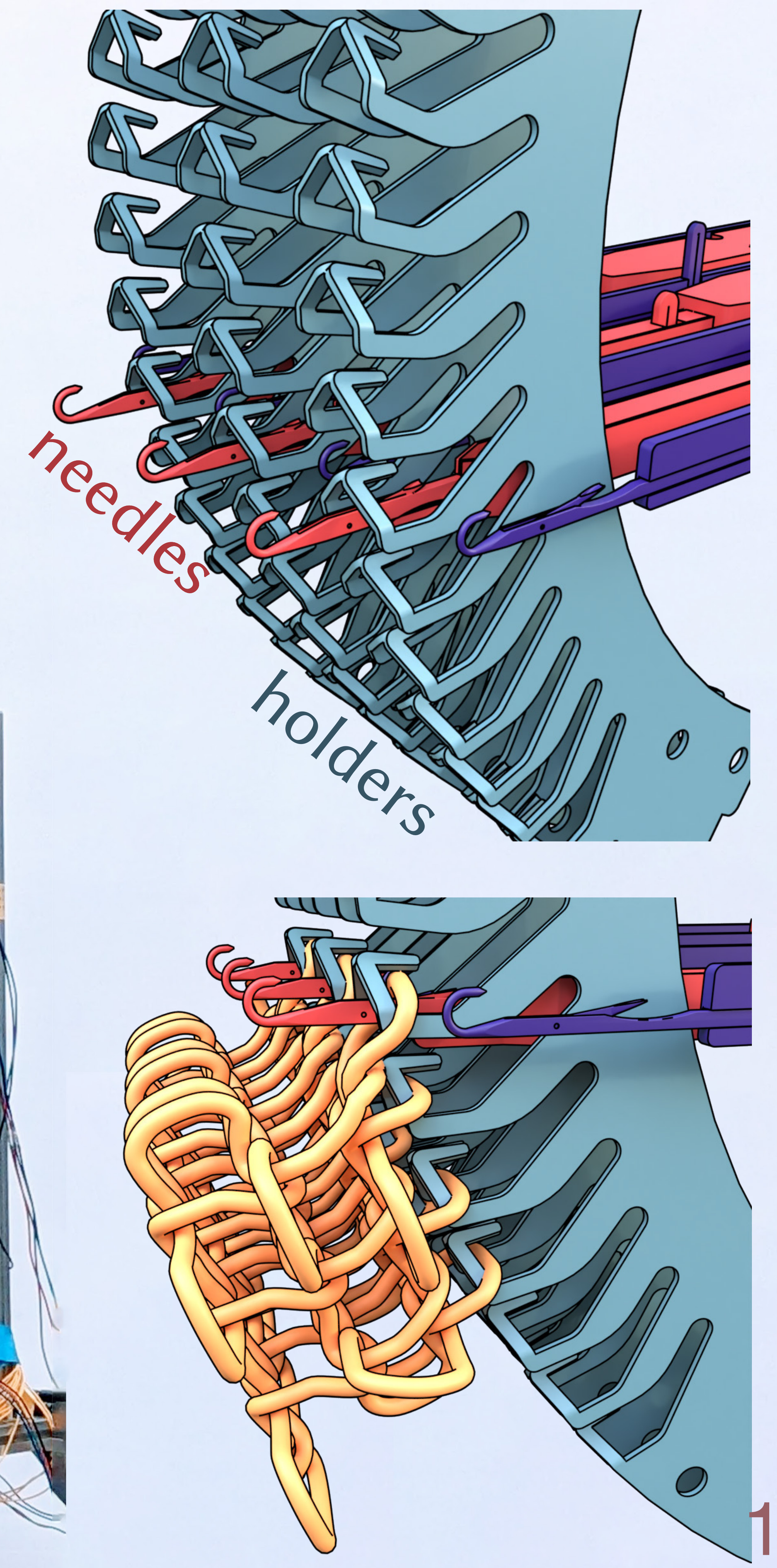


Our solid knitting machine





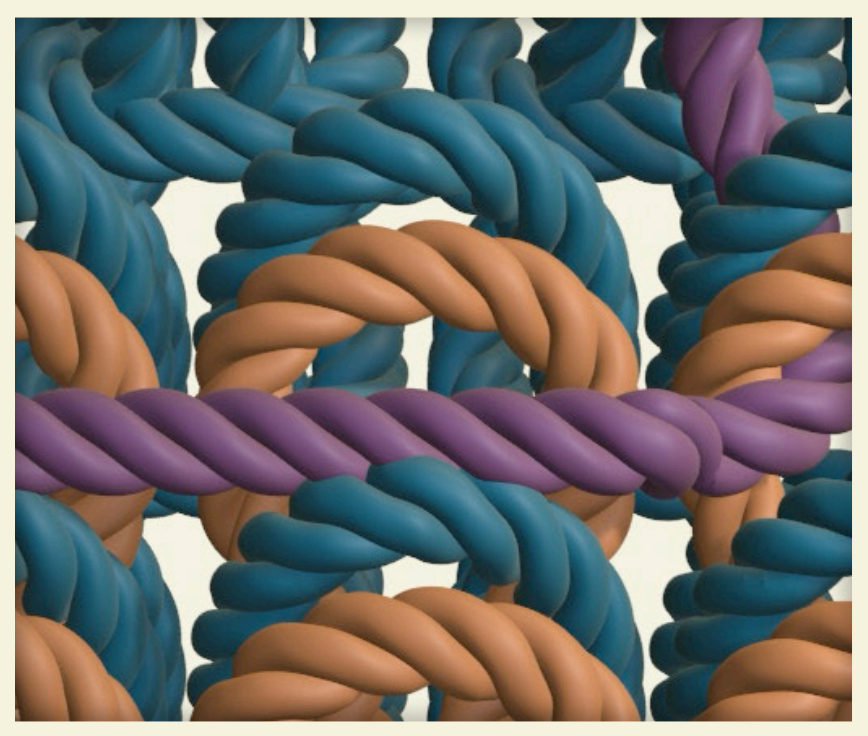





40x

Solid knitting a stitch

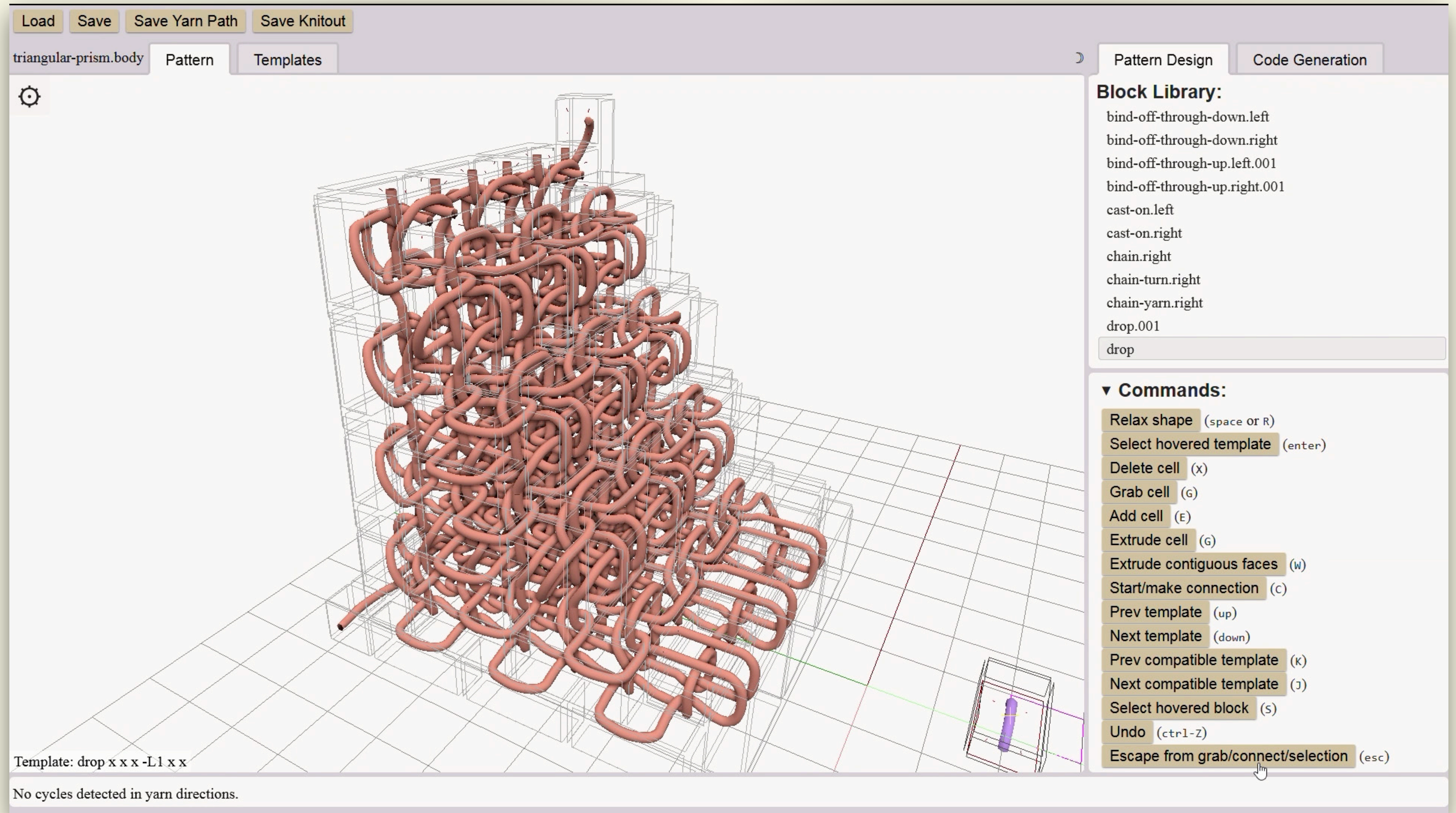
Recall: pull a new loop through two existing loops



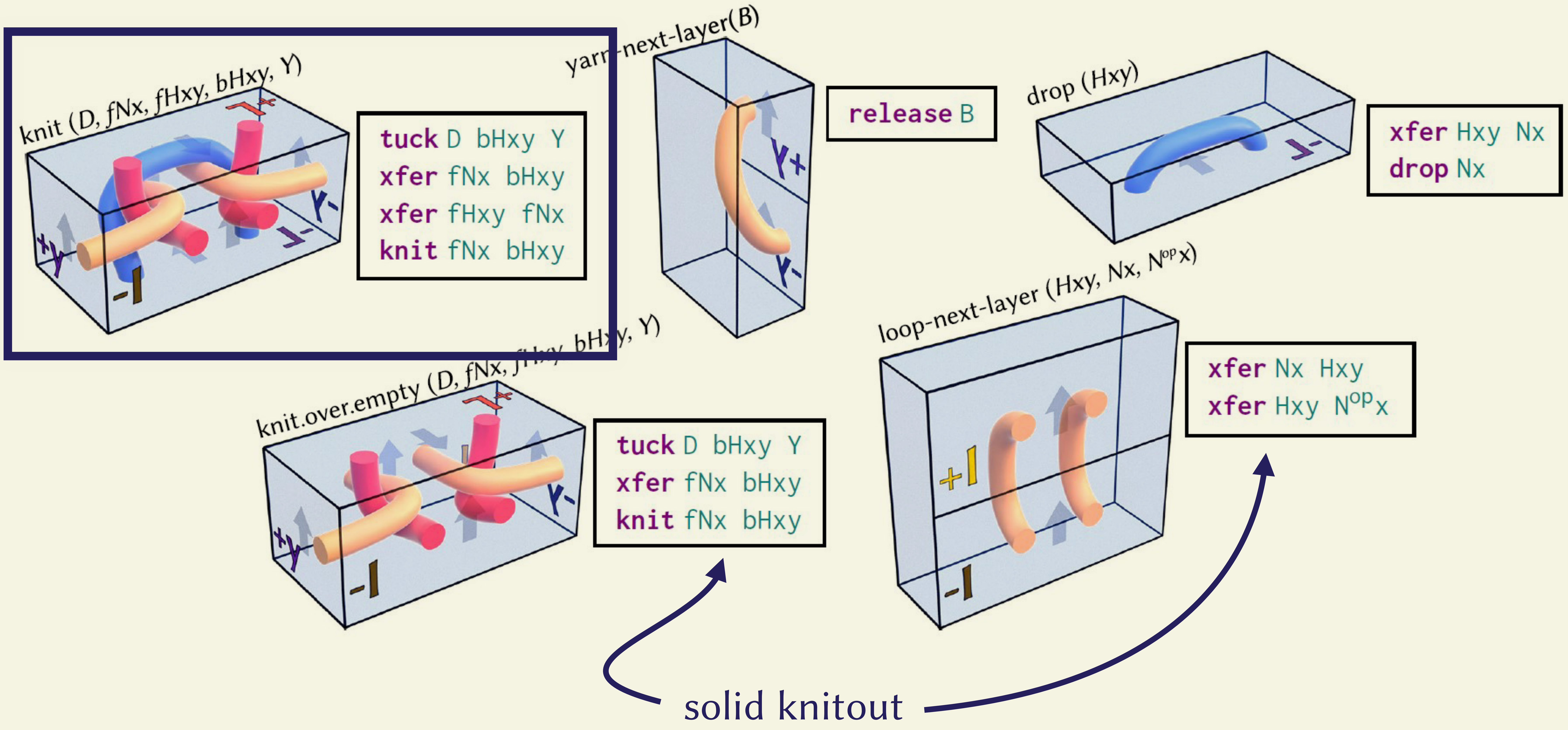
completed a new stitch


16x
1
Tuck
*Mirrored

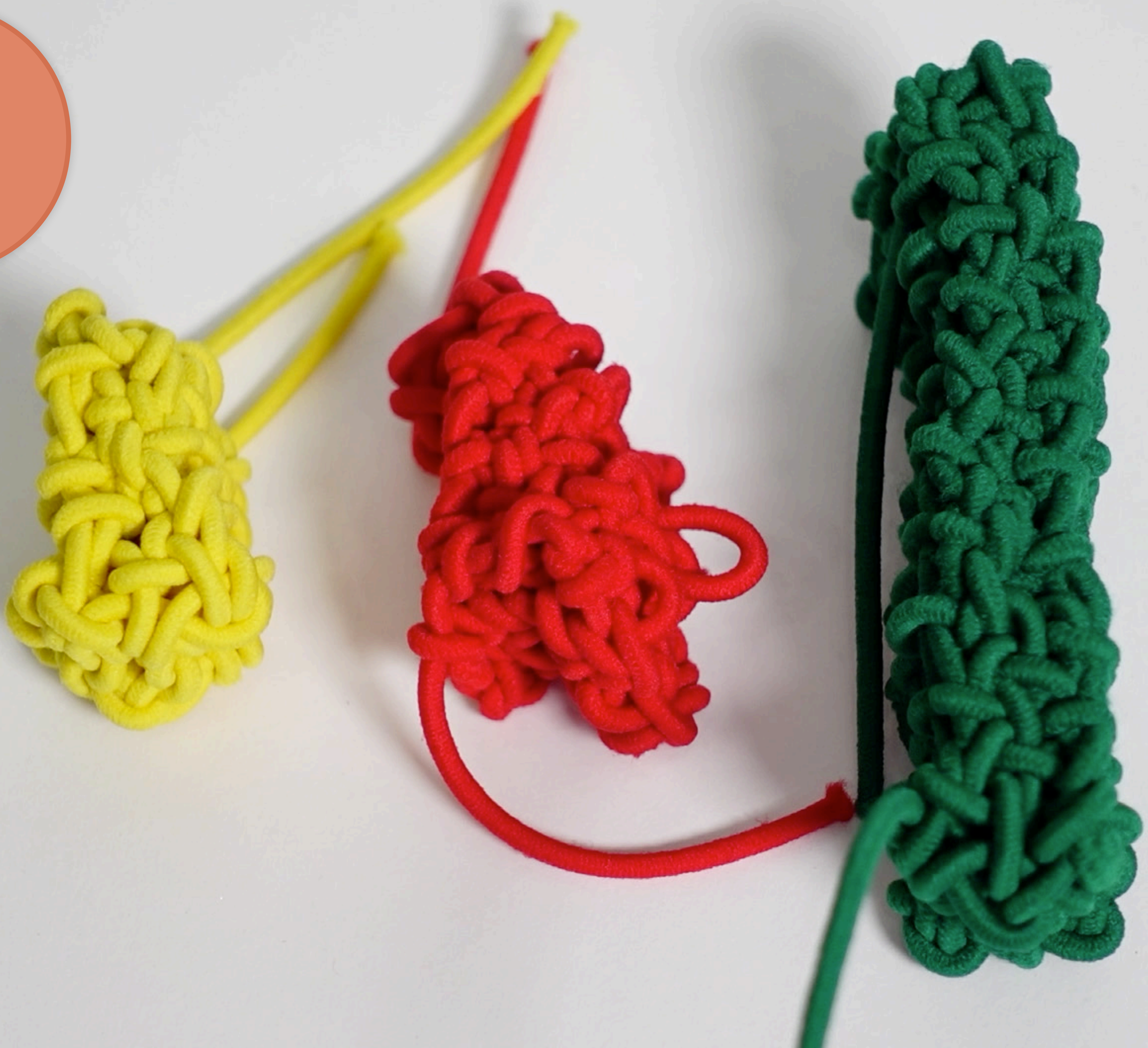
Design tool


Load
Save
Save Yarn Path
Save Knitout
triangular-prism.body
Pattern
Templates
Pattern Design
Code Generation
Block Library:
bind-off-through-down.left
bind-off-through-down.right
bind-off-through-up.left.001
bind-off-through-up.right.001
cast-on.left
cast-on.right
chain.right
chain-turn.right
chain-yarn.right
drop.001
drop
▼ Commands:
Relax shape (space or R)
Select hovered template (enter)
Delete cell (X)
Grab cell (G)
Add cell (E)
Extrude cell (G)
Extrude contiguous faces (W)
Start/make connection (C)
Prev template (up)
Next template (down)
Prev compatible template (K)
Next compatible template (J)
Select hovered block (S)
Undo (ctrl-Z)
Escape from grab/connect/selection (esc)
Template: drop x x x -L1 x x
No cycles detected in yarn directions.

Augmented stitch volumes

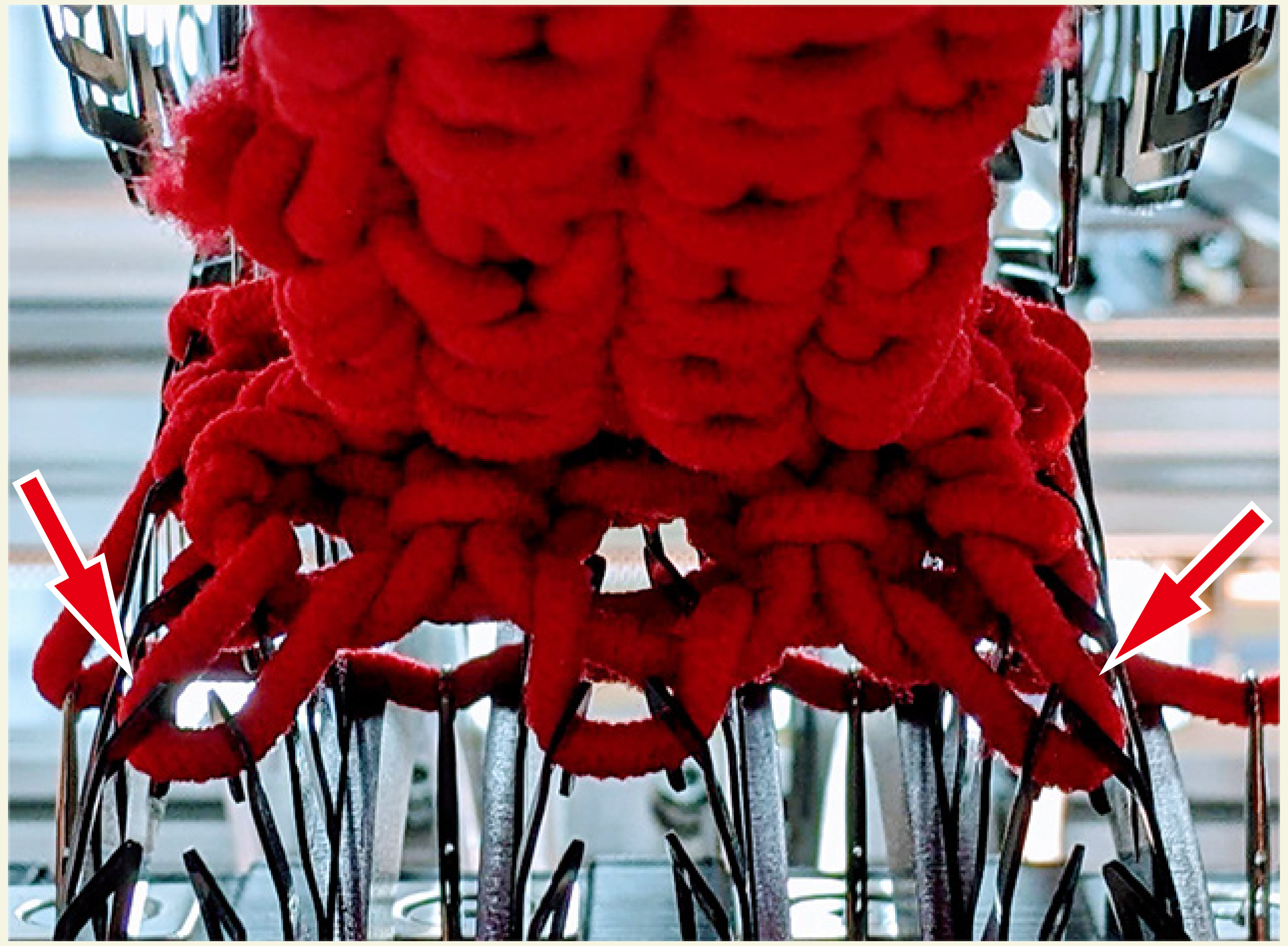


Results

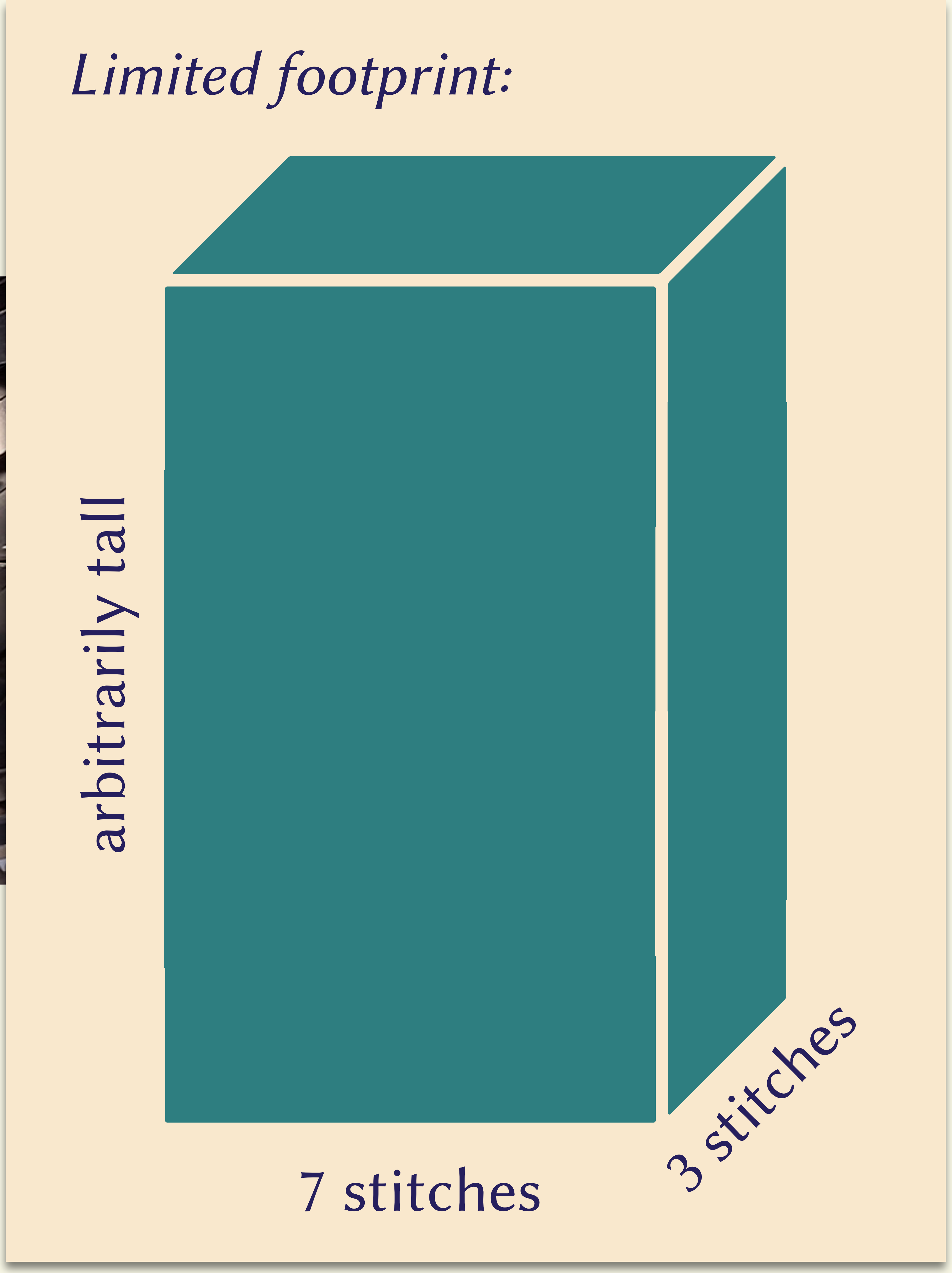


Future work: mechanical improvement

Loop shape consistency


(bottom view)


(Side View)



Future work: physical properties

[smart-knit-crocheting.com]

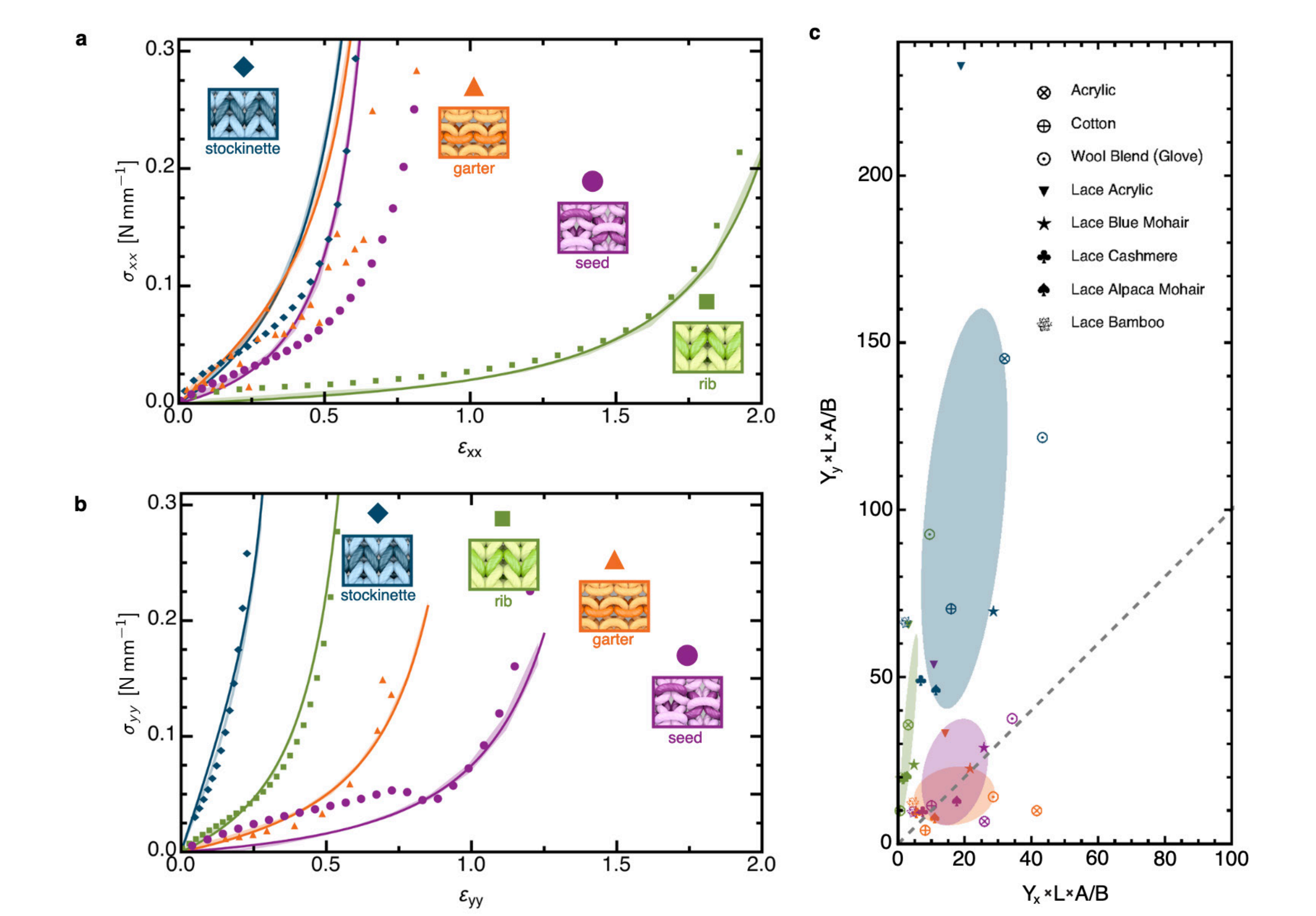
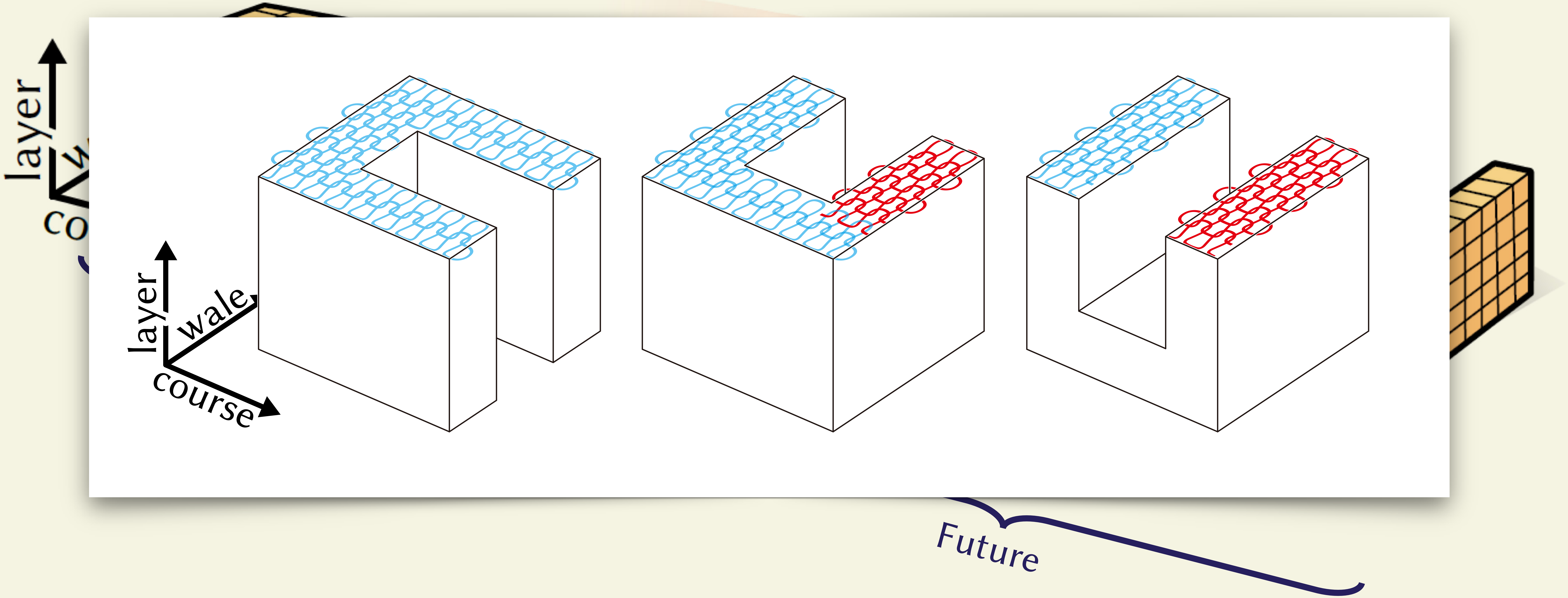


Fig. 2 | Experimental and simulated results of uniaxial stretching. The stress-versus-strain relations for the four fabrics made from the acrylic yarn in the (**a**) *x*- and (**b**) *y*-directions. All of the data for each type of fabric is displayed by a different color: stockinette in blue, garter in orange, rib in green, and seed in purple. The experimental data is shown in the translucent regions where the width of the region is one standard deviation of the four experiment runs. The simulation data is shown with solid symbols. The solid curves are fits to the constitutive relations. This is a system where the linear response for each fabric is significantly different despite only small differences in the stitch configuration, whereas the nonlinear parts are quite similar. Experiments applying force in the *x*-direction show the extreme extensibility of the rib pattern compared with the other three. Garter and seed dominate in the *y*-direction. Note, the experimental measurements for seed fabric differ from that of simulations due to a compression-related buckling instability in the computation, investigated in Supplementary Note 4 and Supplementary Fig. 8. **c** Normalized rigidity plot of all fabric samples, where Y_i is the Young's modulus in the i^{th} direction in N/mm (Supplementary Tables 10, 12, 14, 20), L is the length of yarn per stitch in mm (Supplementary Tables 2, 3, 6), A is the area of one stitch in mm^2 (Supplementary Tables 5, 6), and B is the bending modulus in N mm^2 (Supplementary Table 1). The colored ellipses represent one standard deviation for each of the four types of fabric and are oriented along the principal axes. The gray dashed line represents an isotropic mechanical response. The same analysis was conducted on the un-normalized rigidities, shown in Supplementary Fig. 11. Source data are provided as a Source Data file.

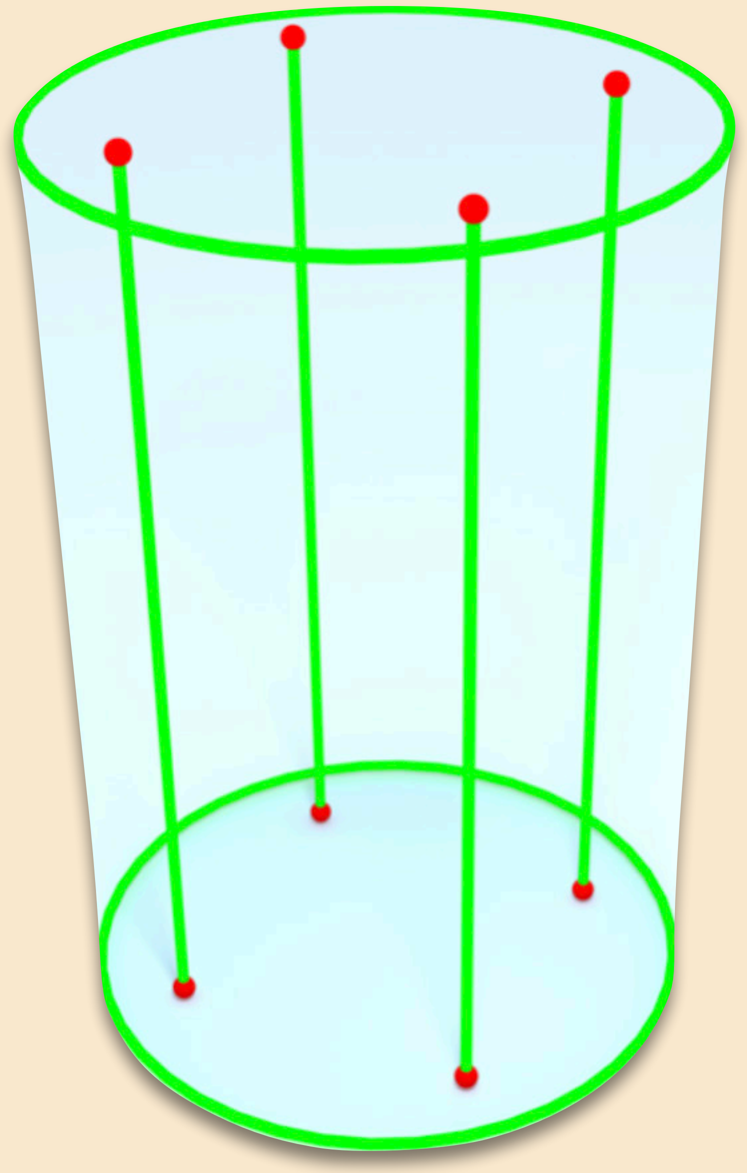
[Singal *et al.* 2023]

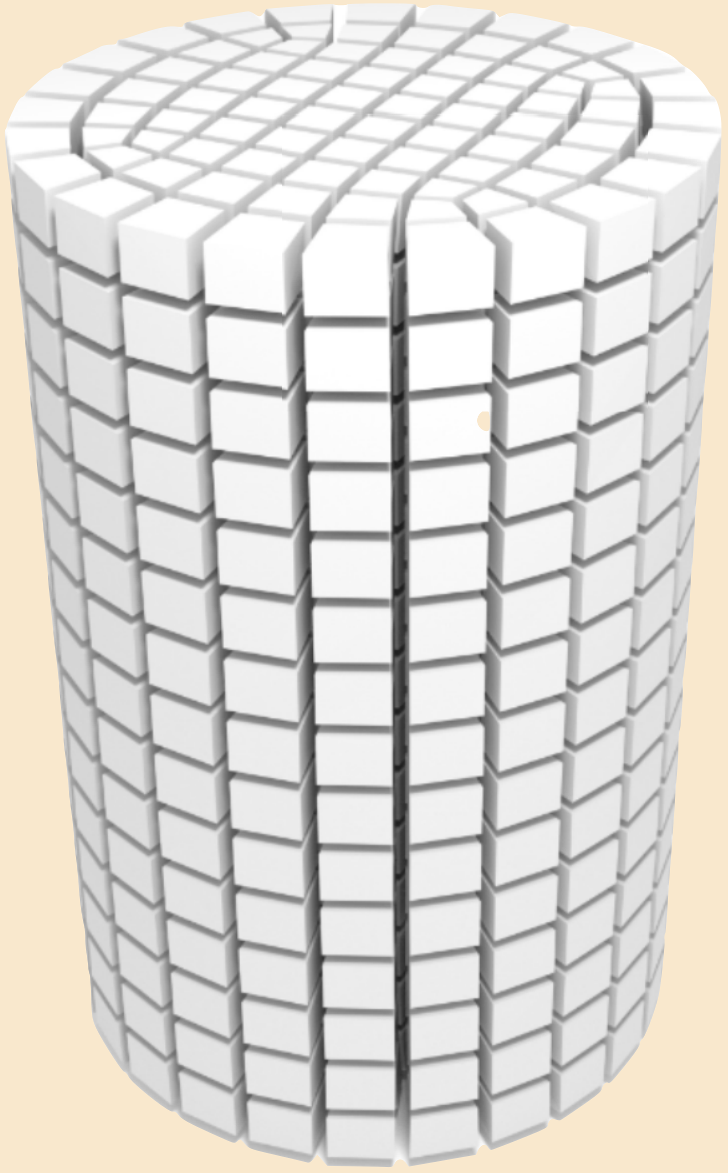
Future work: machine support for more complex shapes



Future work: shaping

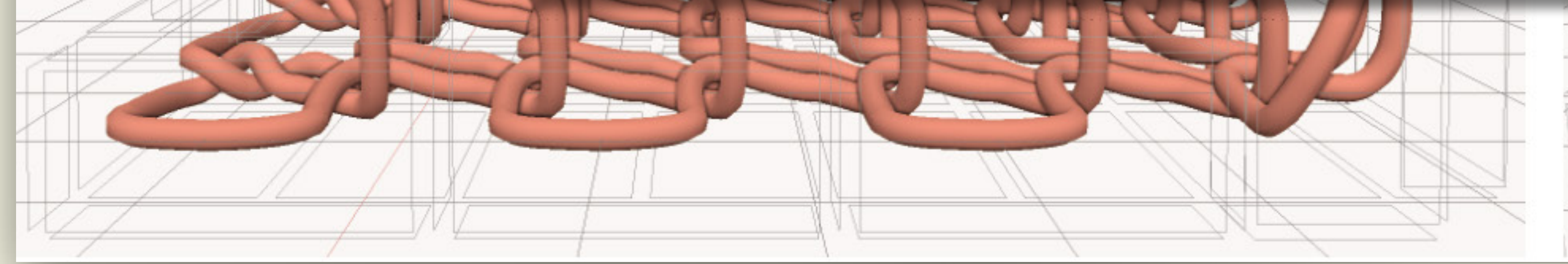
Idea: use *hexahedral meshing*

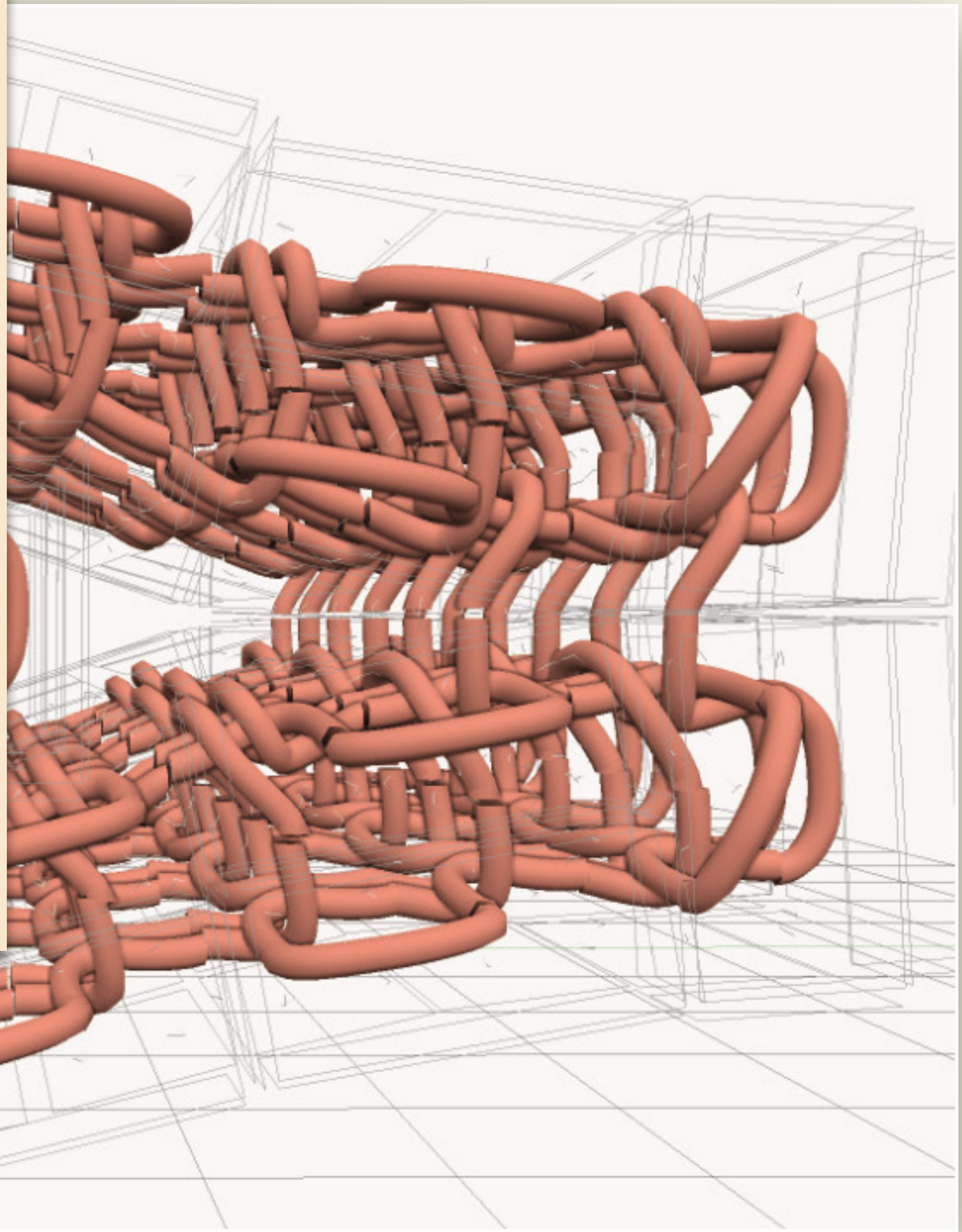




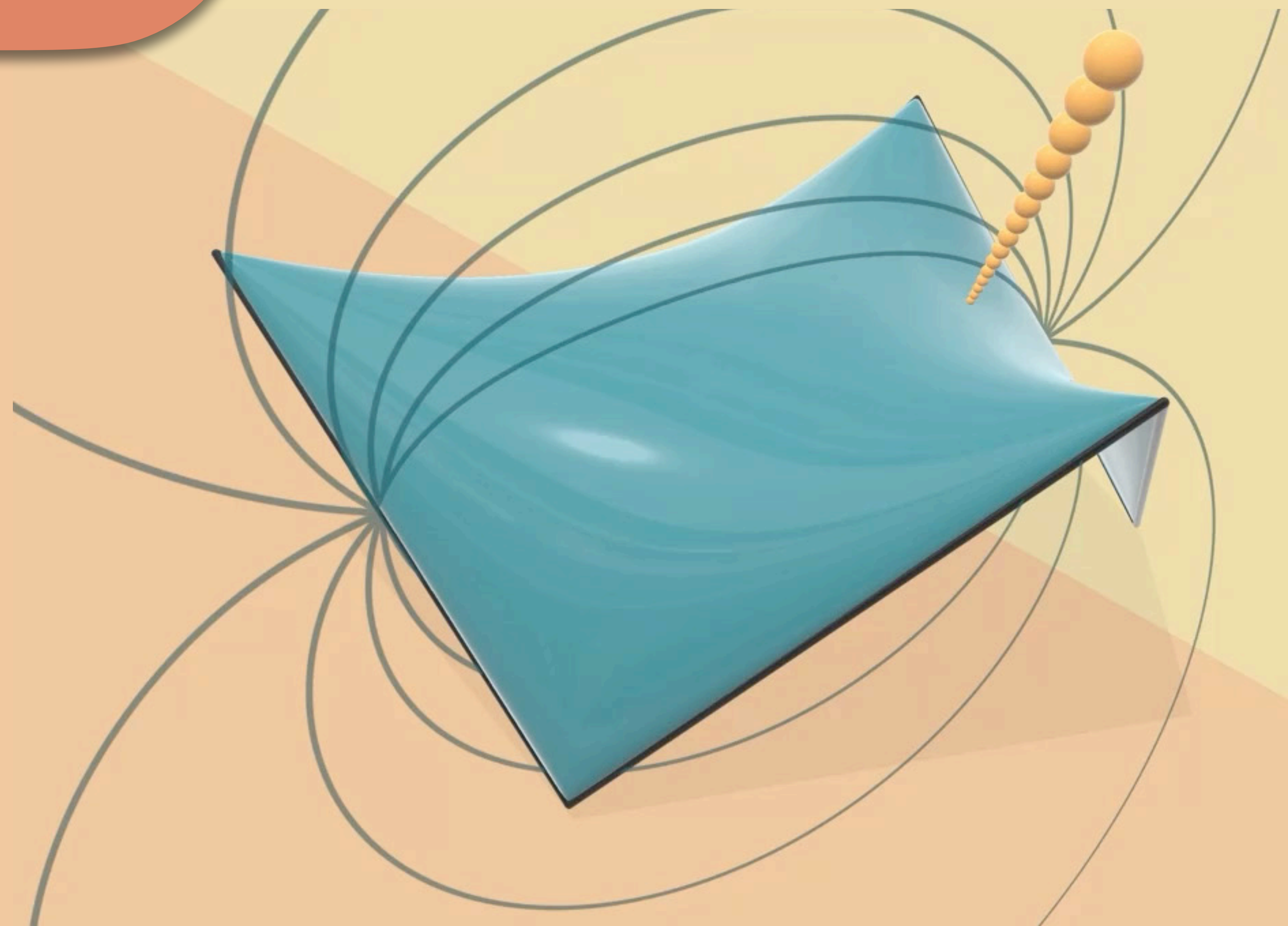
[Liu *et al.* 2018]

with later layers





Harmonic Hitting



Harmonic functions

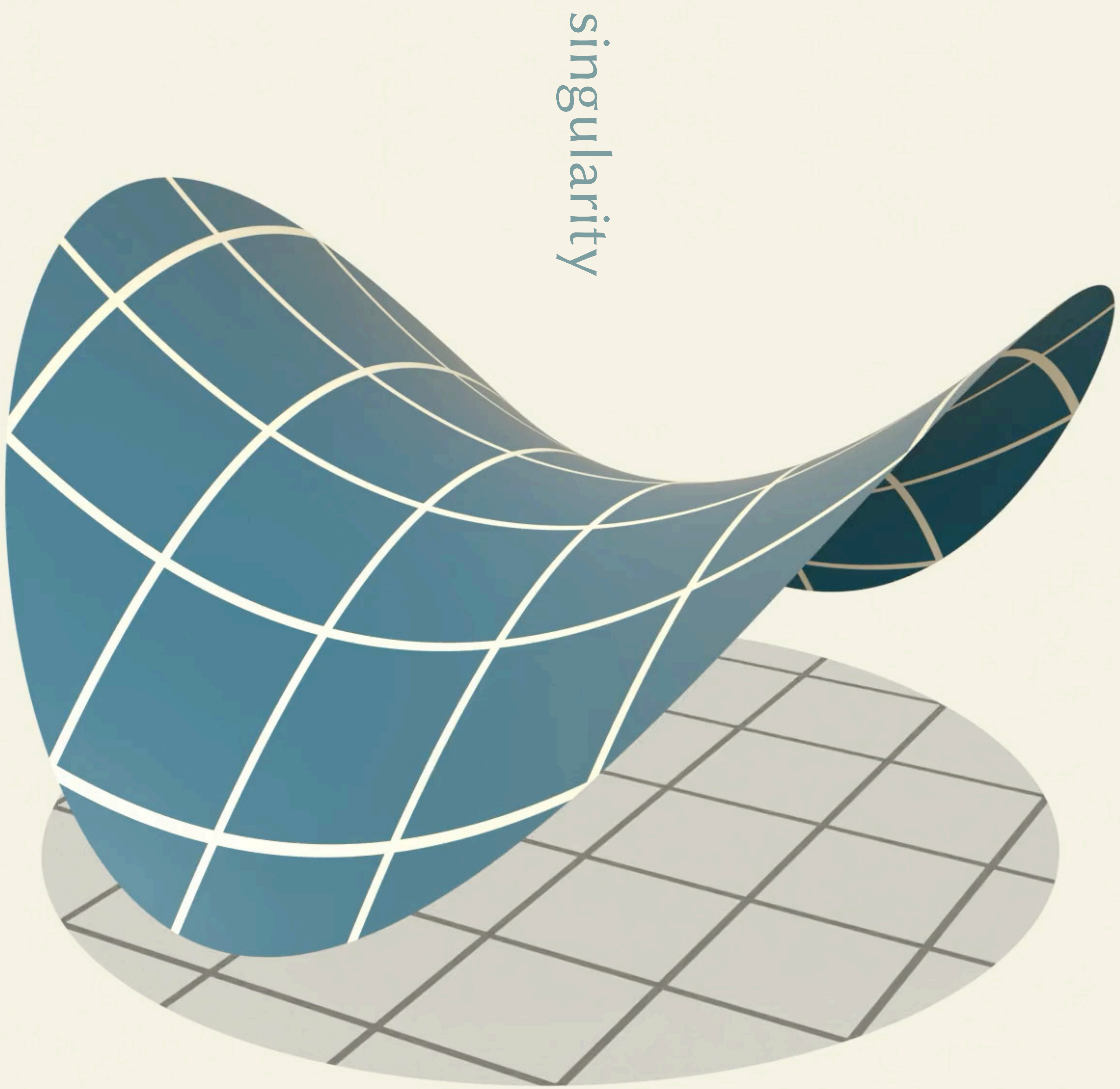
special kind of function

$$\Delta f := \sum_i \frac{\partial^2 f}{\partial x_i^2} = 0$$

well-understood mathematically

Harmonic functions

$$\Delta f := \sum_i \frac{\partial^2 f}{\partial x_i^2} = 0$$



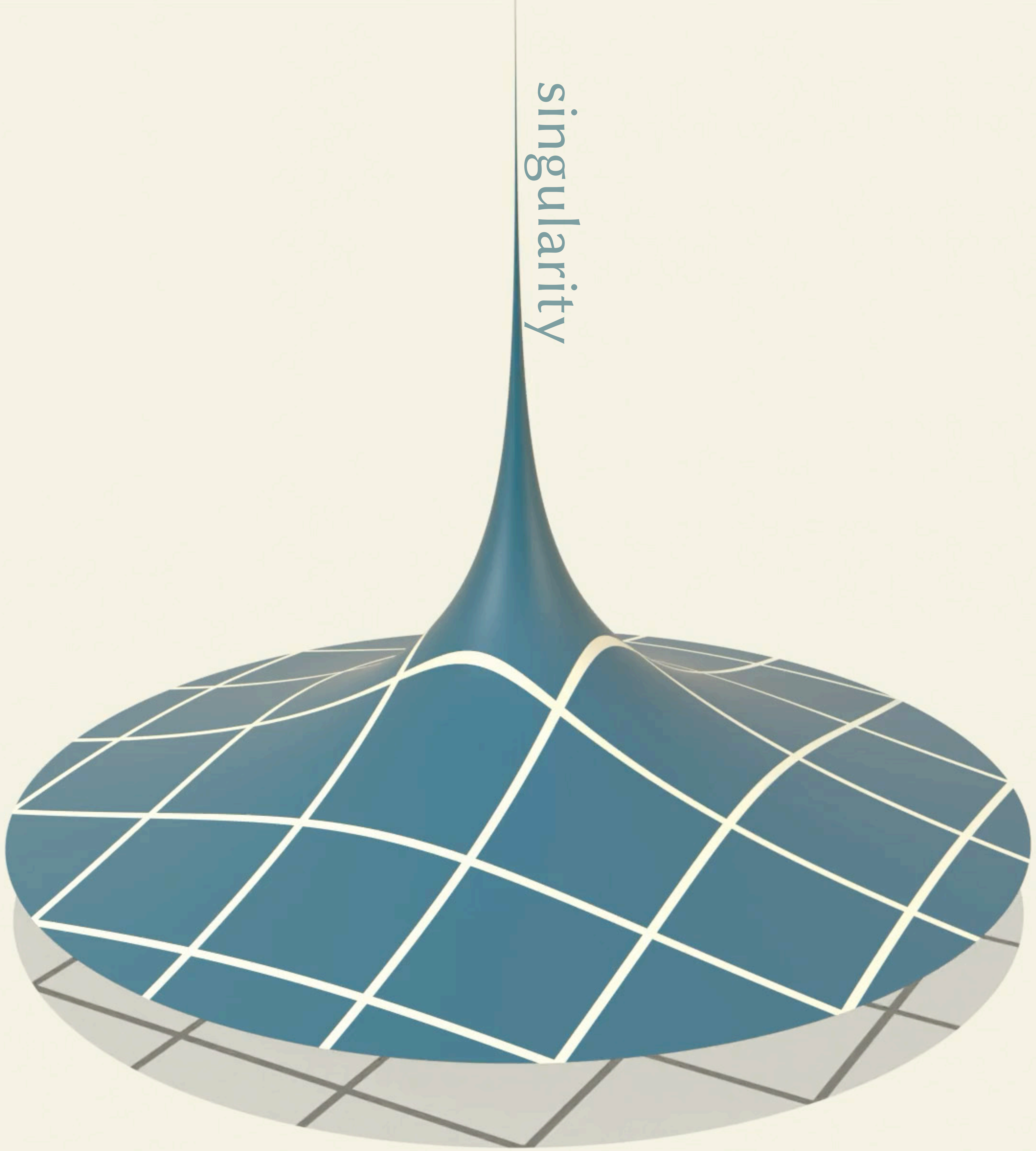
harmonic function

Green's function

$f(x,y) = x^2 - y^2$

Harmonic functions

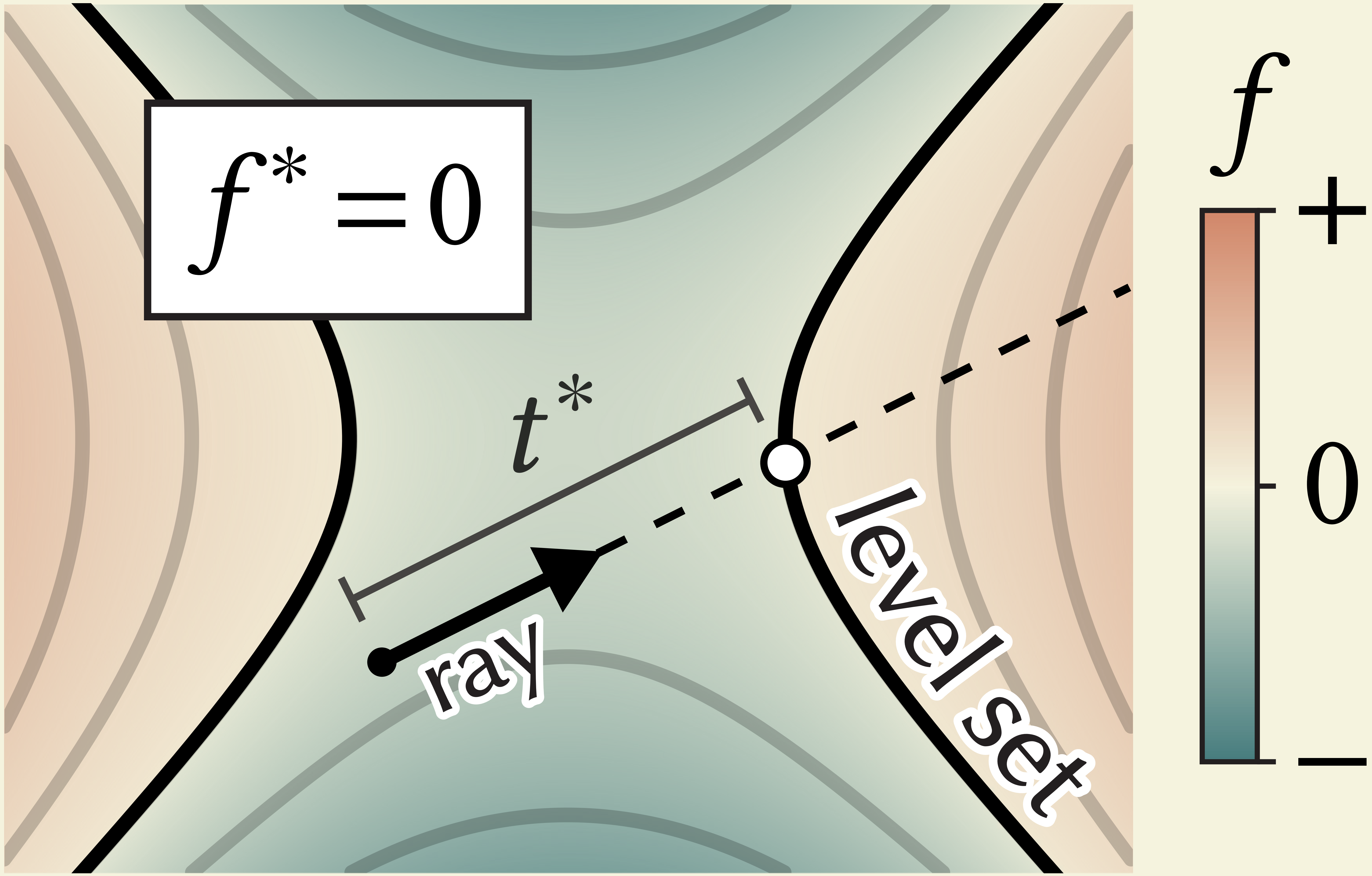
$$\Delta f := \sum_i \frac{\partial^2 f}{\partial x_i^2} = 0$$



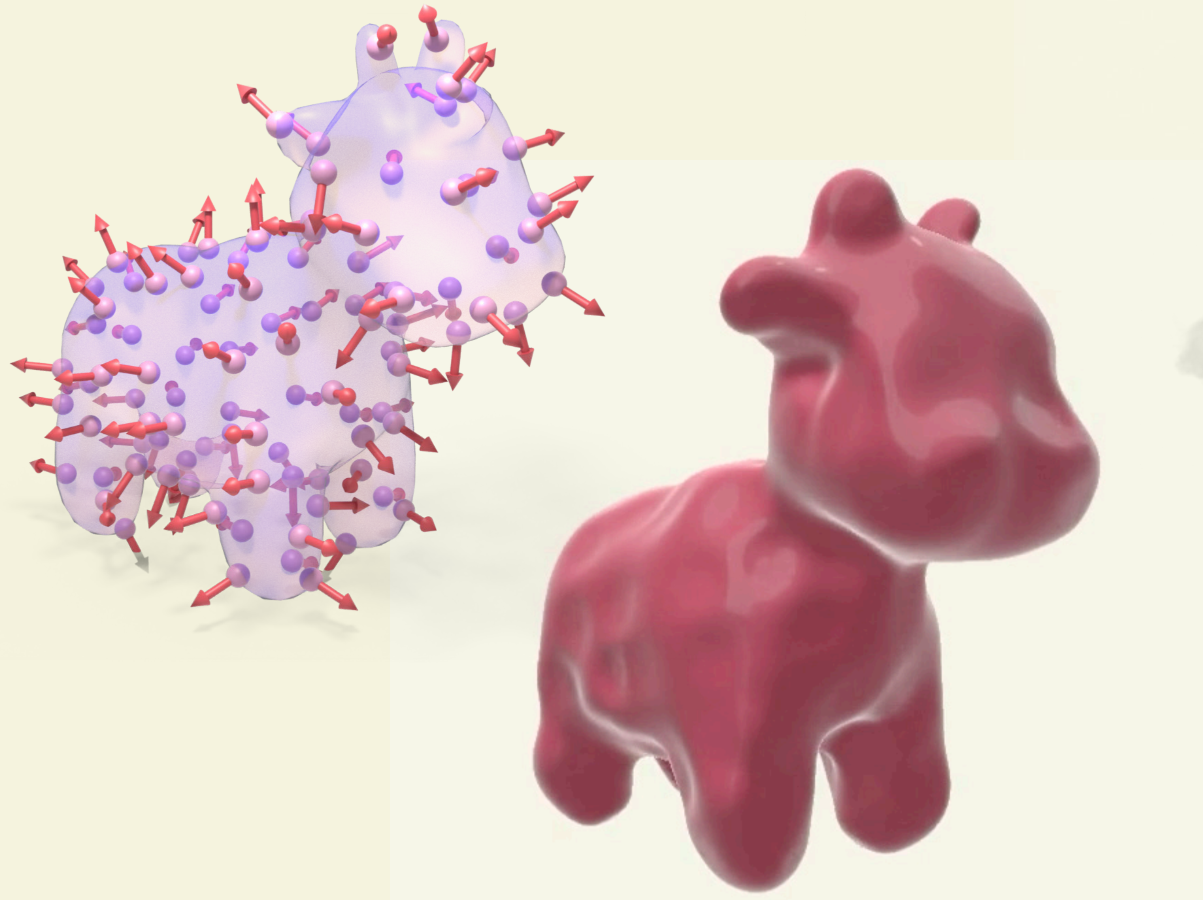
harmonic Greens' function

dipole potential

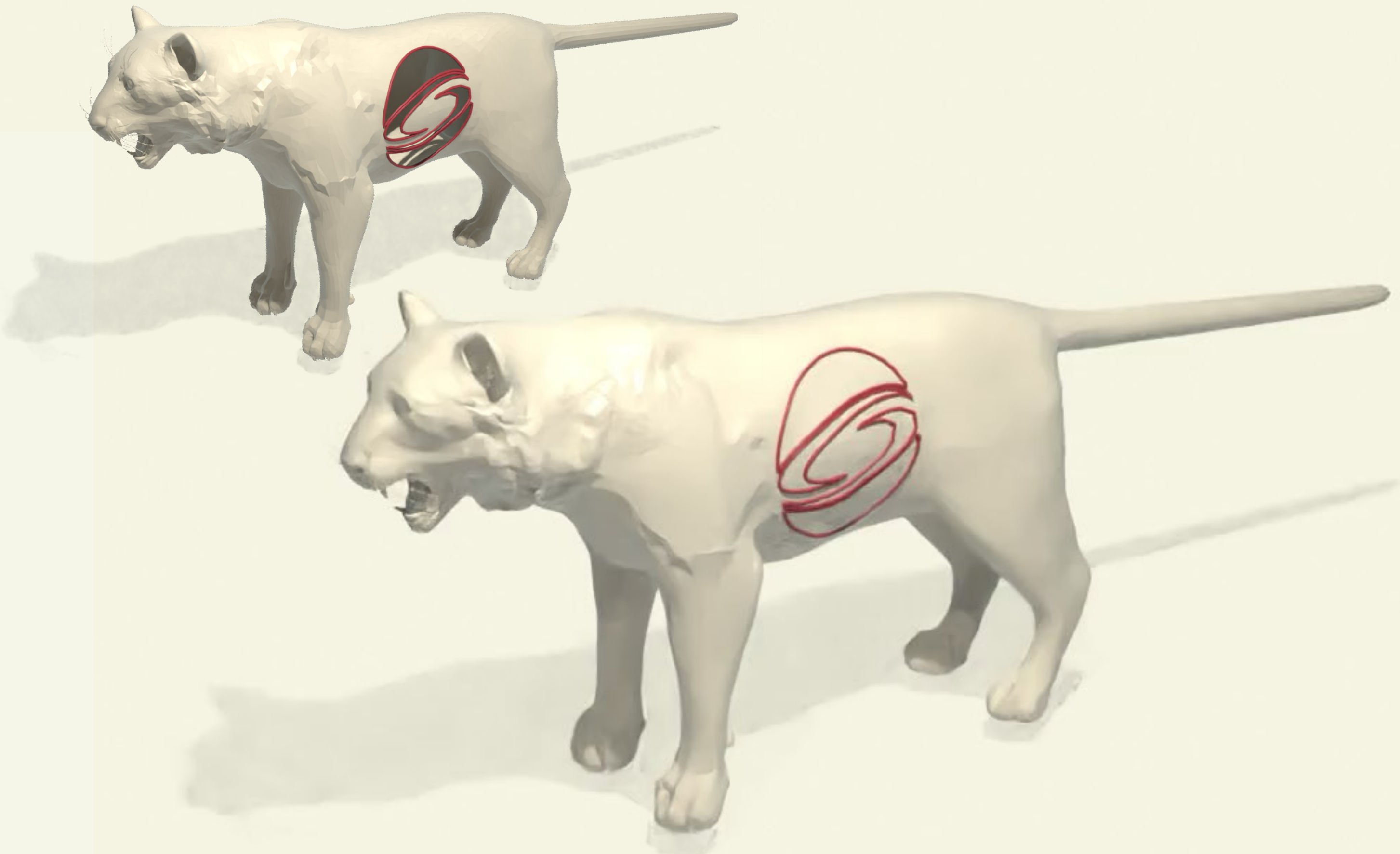
Intersecting a ray with a level set



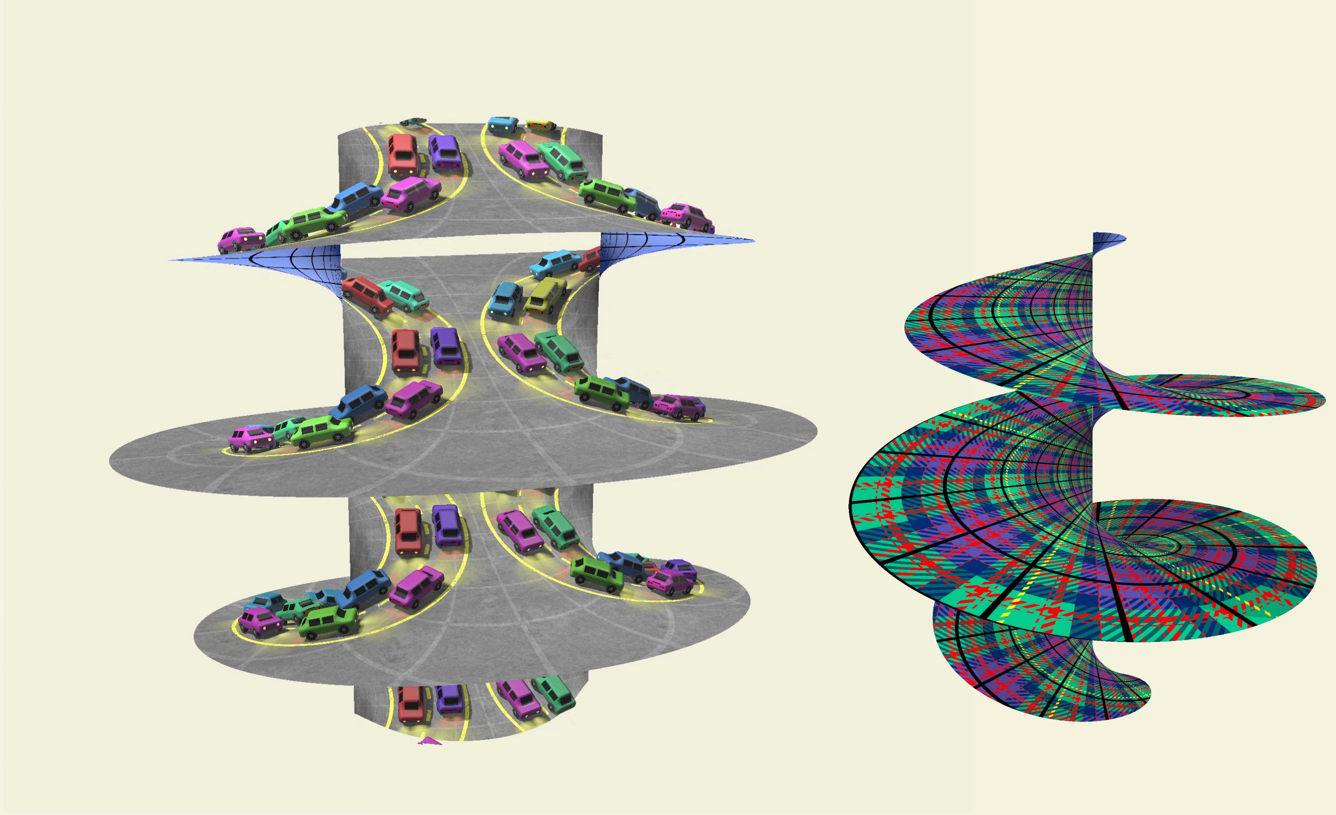
Level sets of harmonic functions show up everywhere



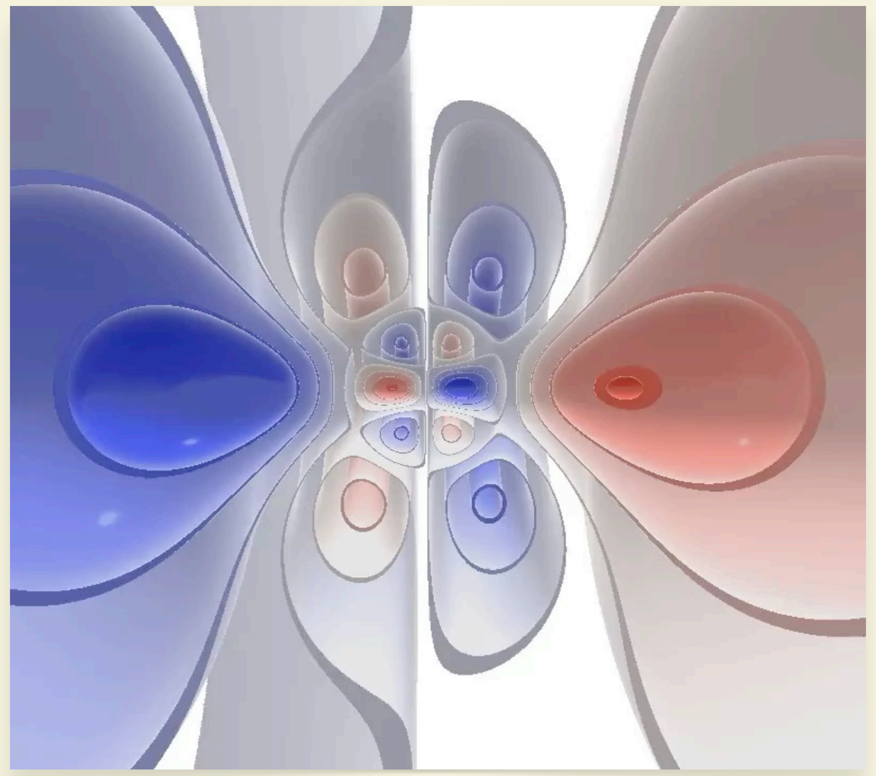
Poisson surface reconstruction
[Kazhdan *et al.* 2006]



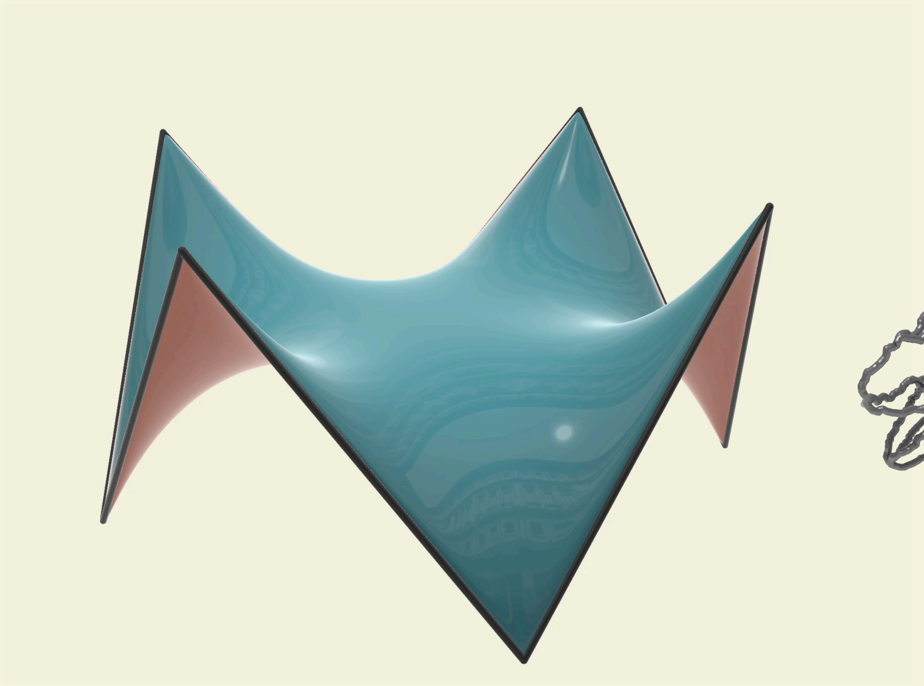
generalized winding numbers
[Jacobson *et al.* 2013]



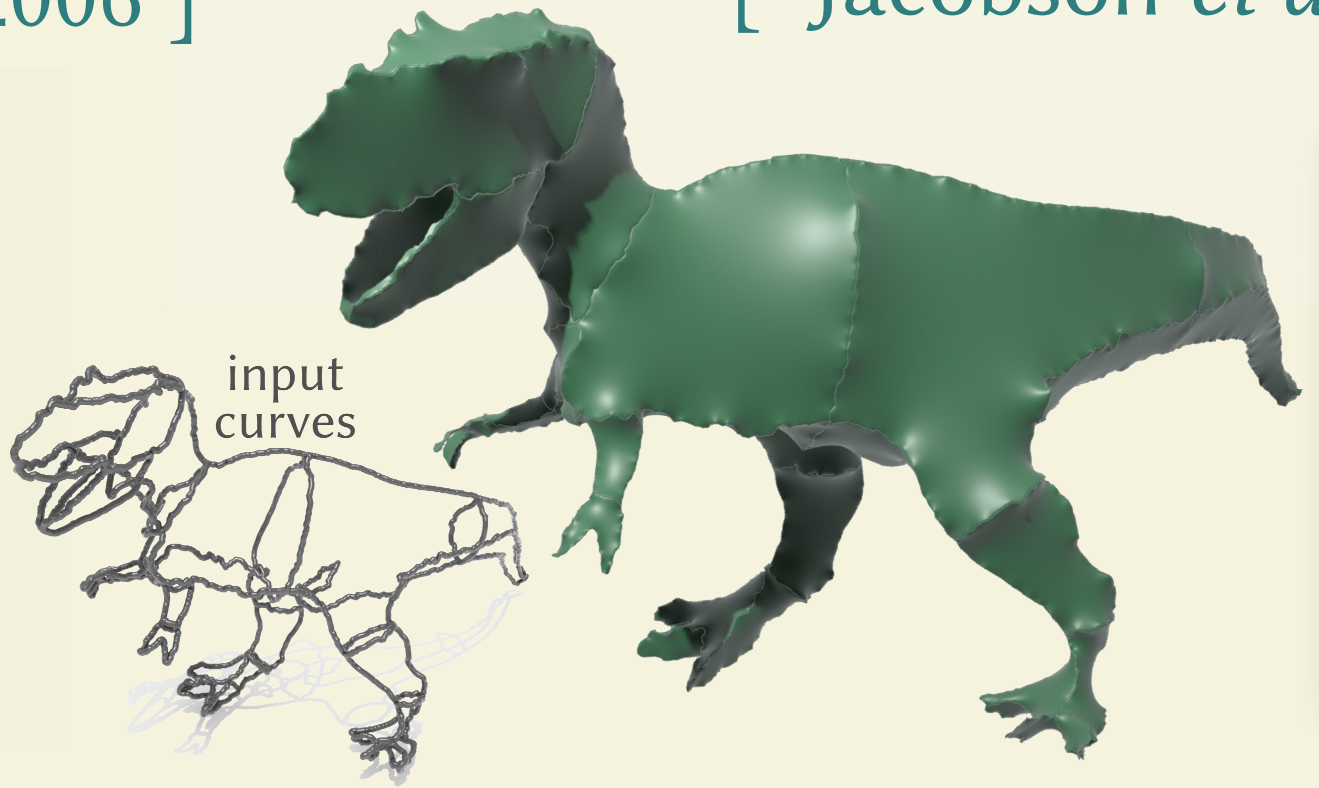
Riemann surfaces
[Riemann 1851]



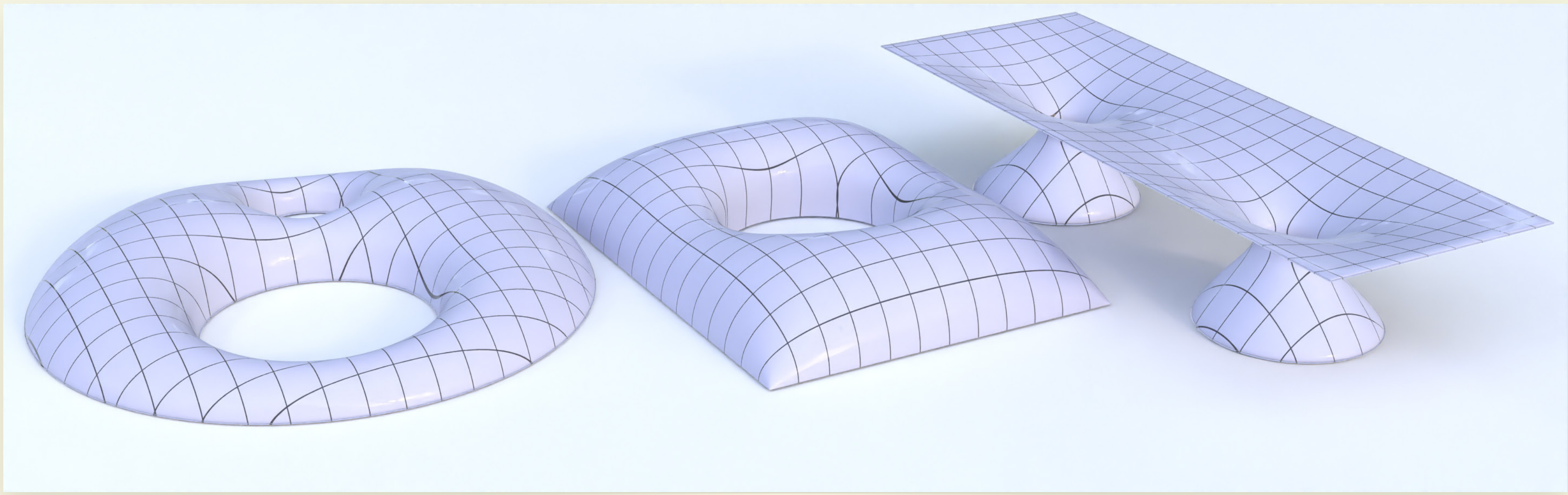
hyperspherical harmonics
[Fock 1935]



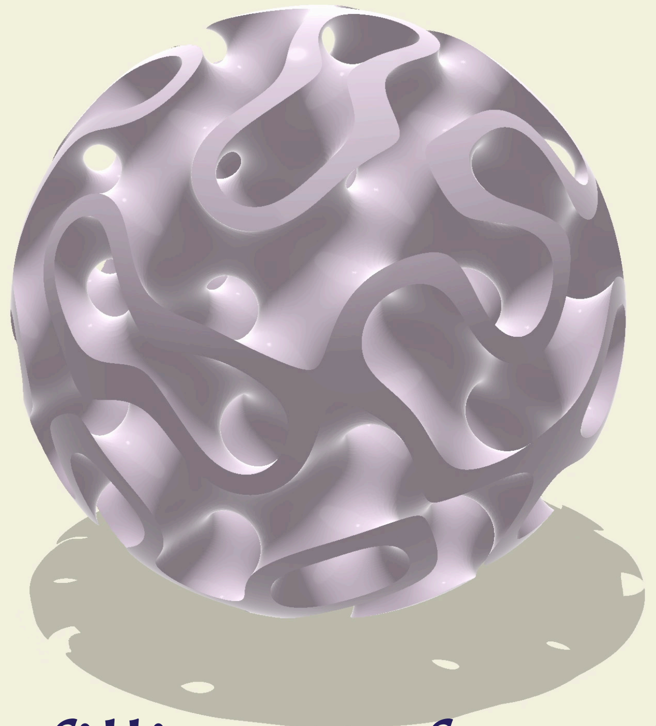
nonplanar polygons
[Maxwell 1873]



curve networks
[de Goes *et al.* 2011]

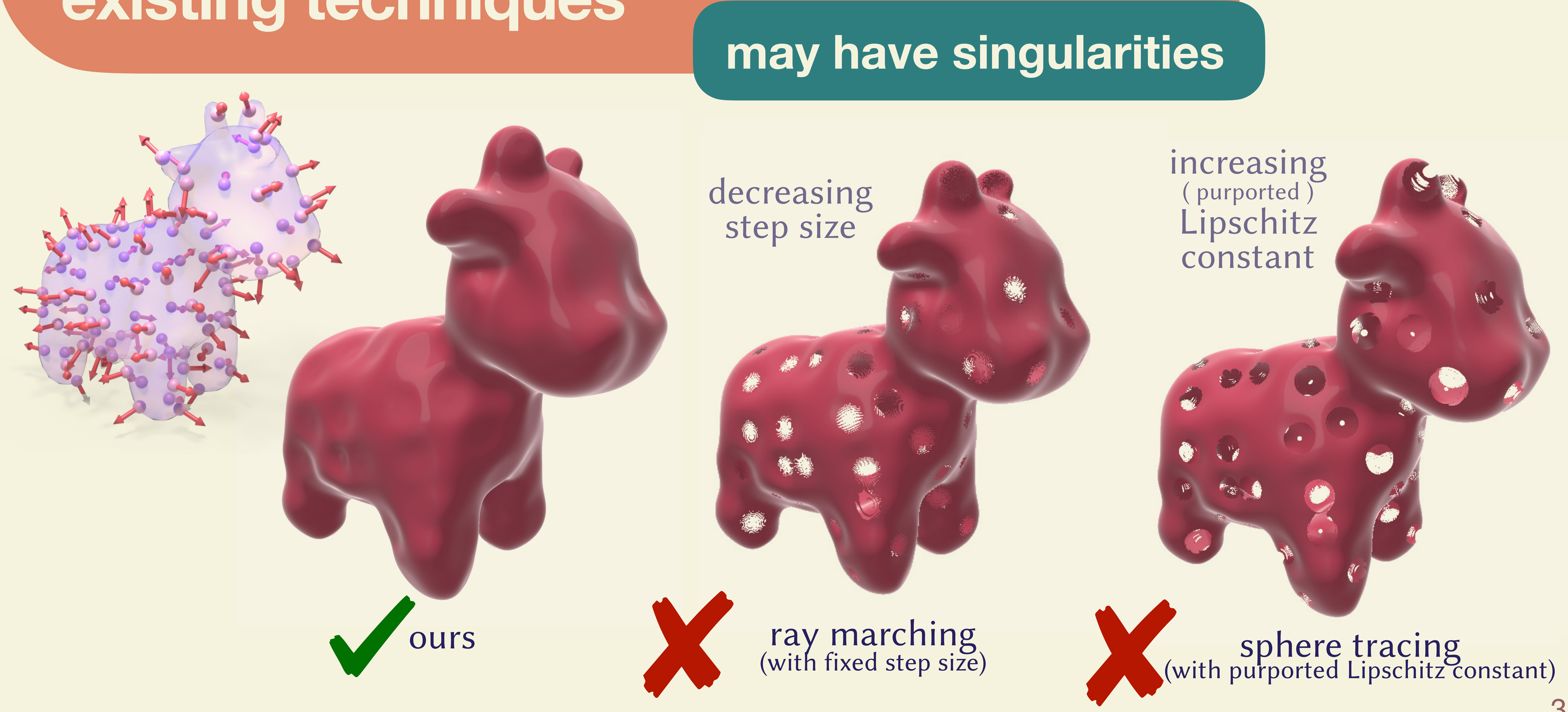


shell structures in architectural geometry [Adiels *et al.* 2022]



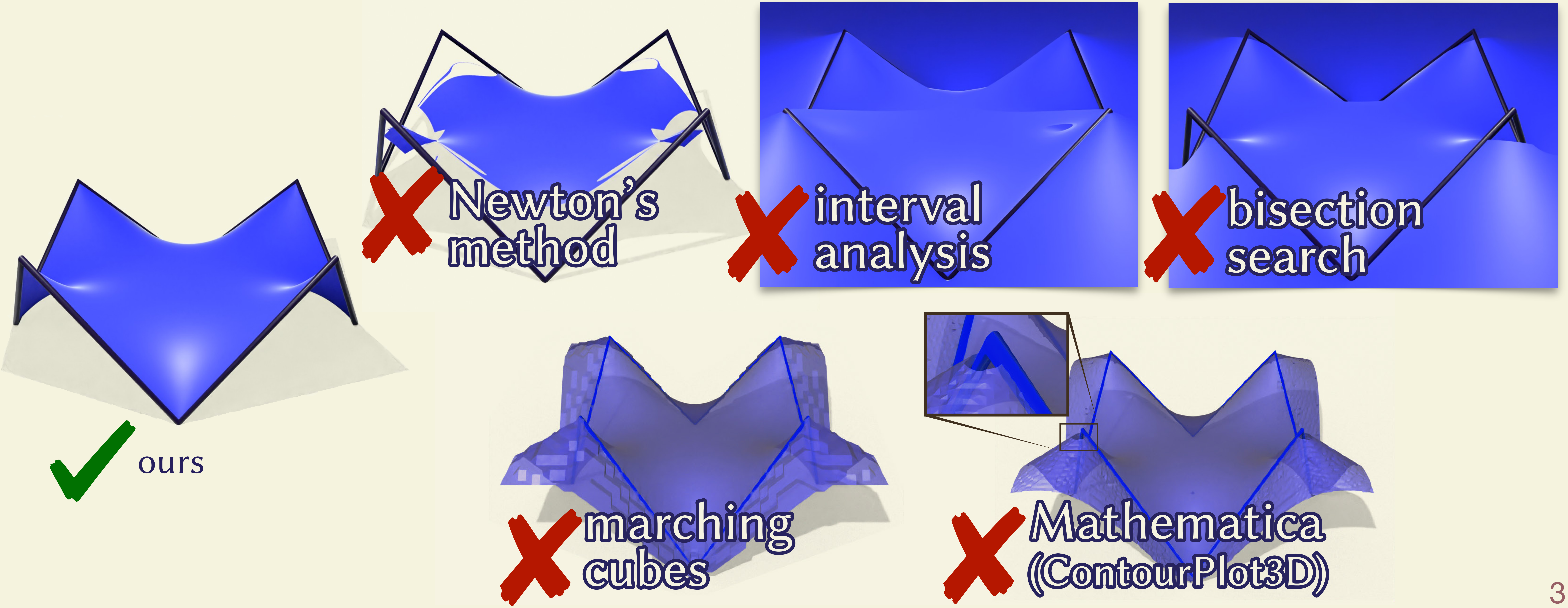
space-filling surfaces for digital fabrication

… but, they're hard to render with existing techniques



… but, they're hard to render with existing techniques

may have boundaries



Sphere tracing

[Hart 1996]

compute intersections for *signed distance functions (SDFs)*

Sphere tracing

[Hart 1996]

compute intersections for *signed distance functions (SDFs)*

Sphere tracing: beyond SDFs
[Hart 1996]

- Easy to generalize to *Lipschitz* functions:

  (essentially, $|\nabla f| \leq L$)

- Important fact:

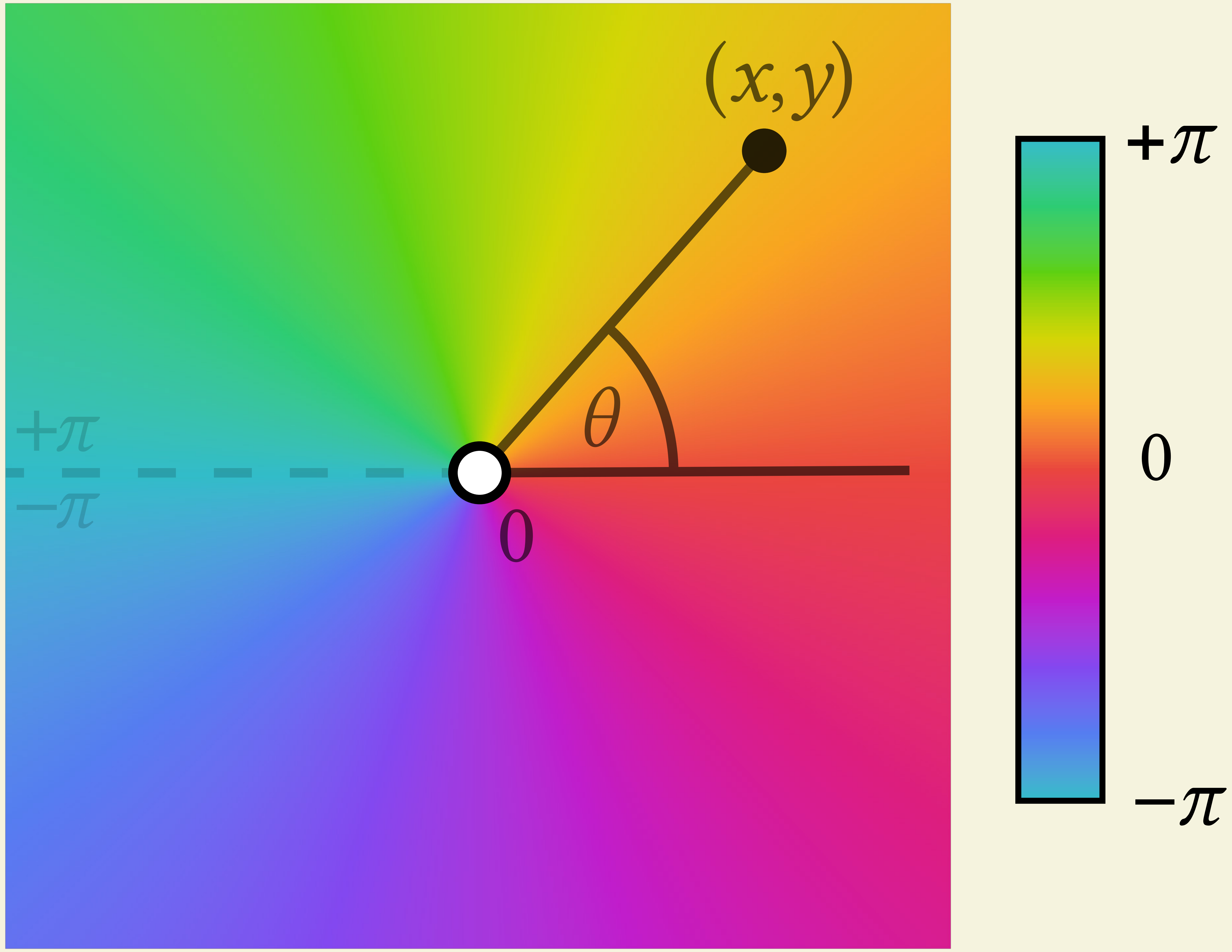
  $$|f(x) - f(y)| \leq L\,|x - y|$$

- provides a conservative bound on distance

go Quilez 2015]

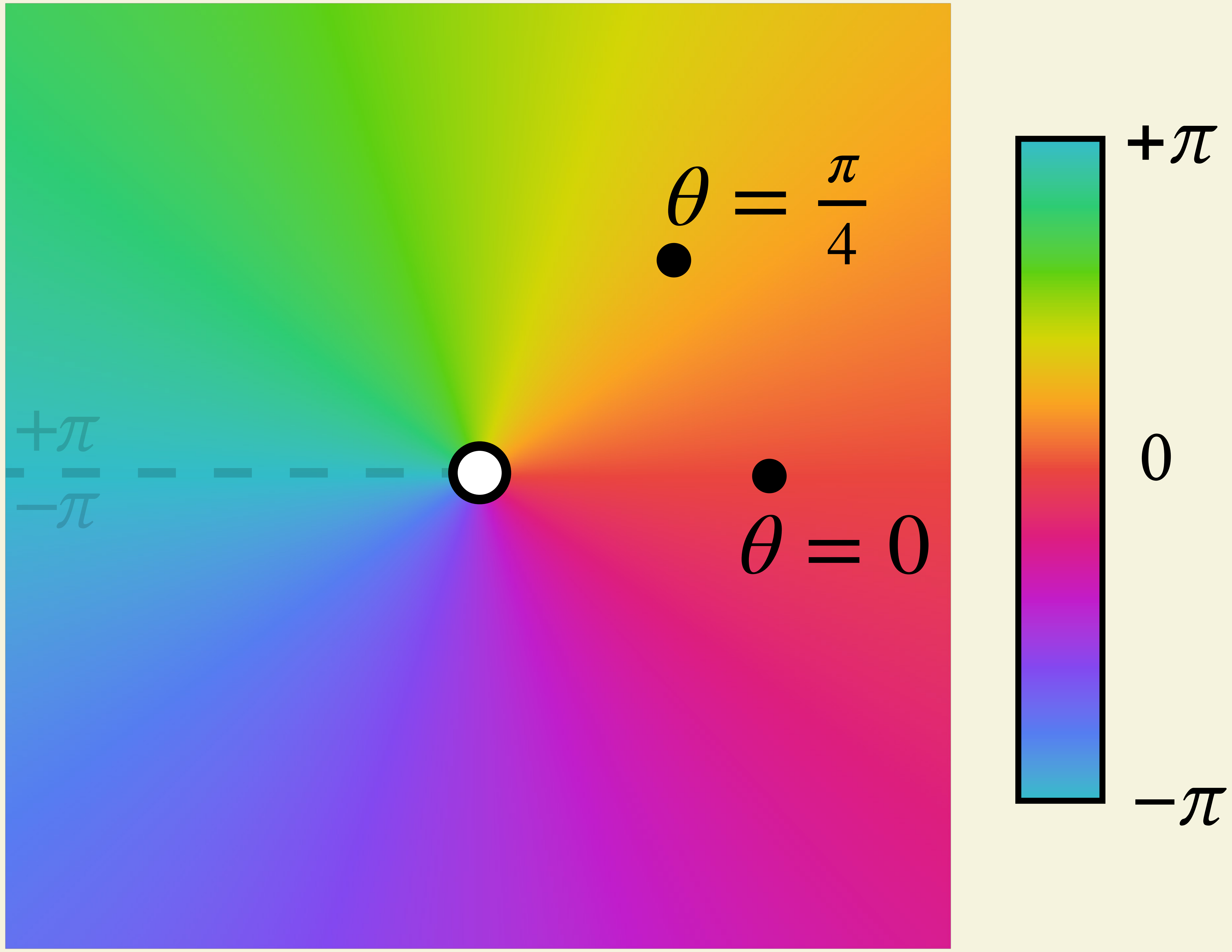
Problem: many harmonic functions are not Lipschitz

$$\theta(x, y) = \text{atan2}(y, x)$$



Problem: many harmonic functions are not Lipschitz

$$\theta(x, y) = \mathrm{atan2}(y, x)$$



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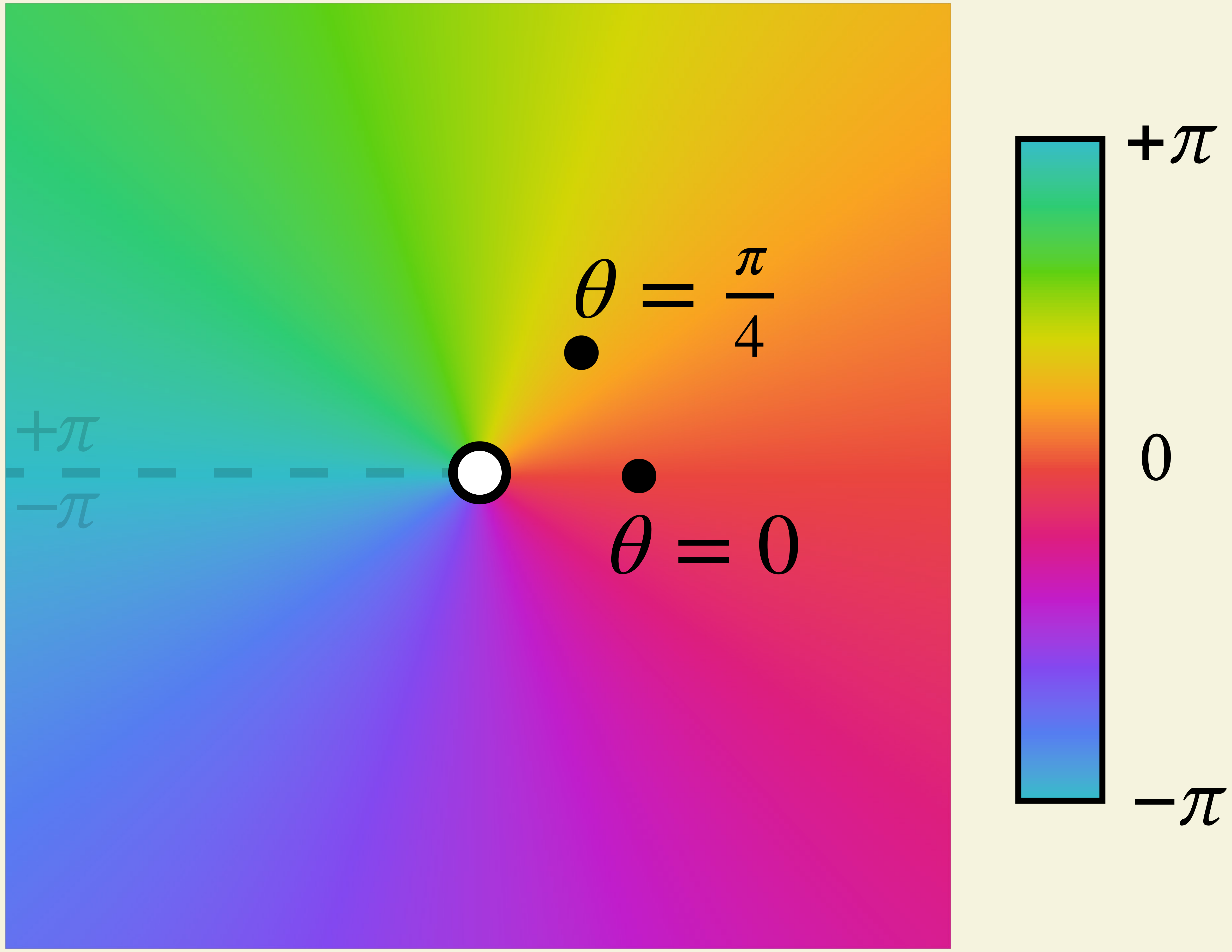
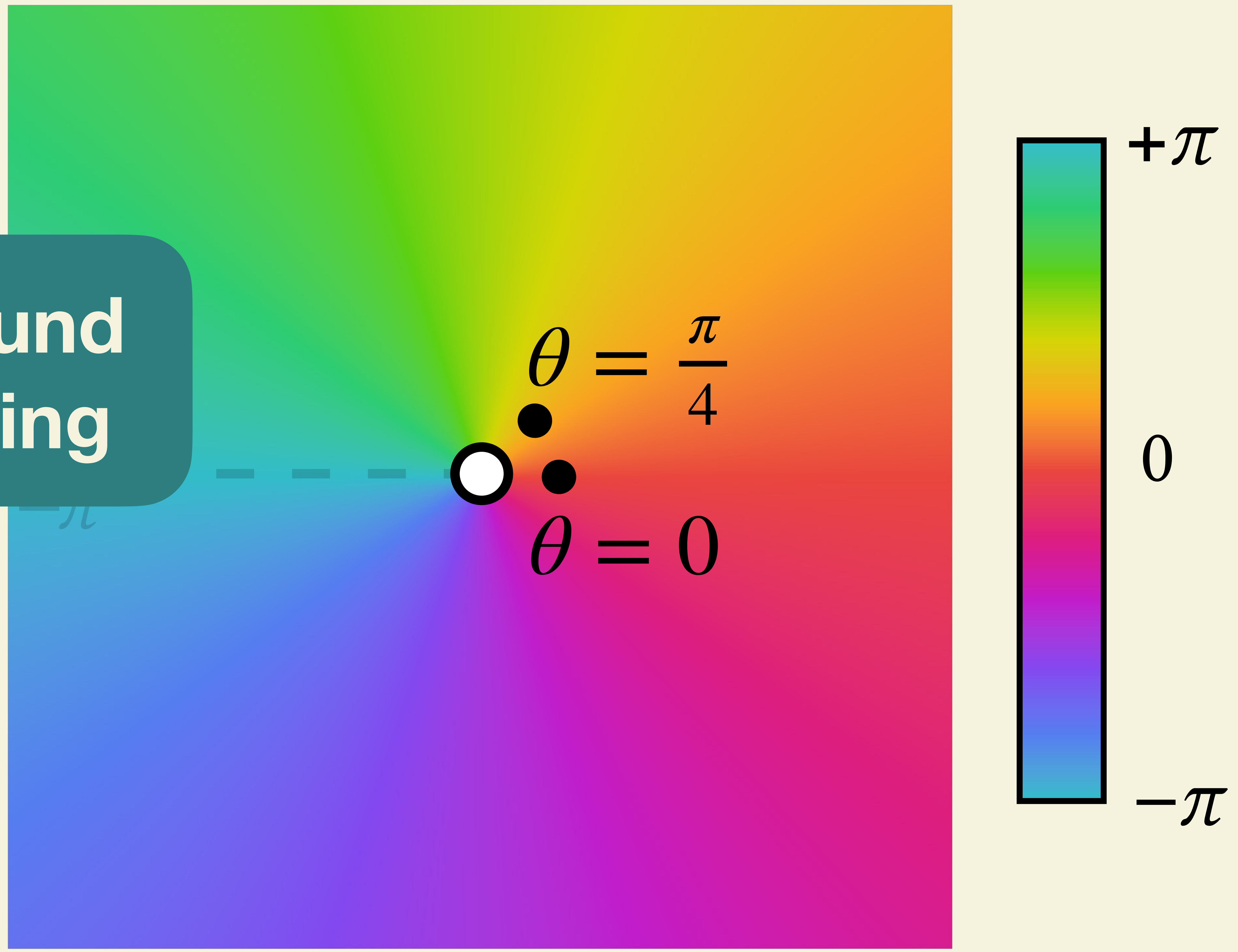


Problem: many harmonic functions are not Lipschitz

$$\theta(x, y) = \mathrm{atan2}(y, x)$$

No matter how close points get, function values never get closer

no distance bound for sphere tracing



Main idea: get distance bounds from Harnack's inequality

Let f be a positive harmonic function on a ball:

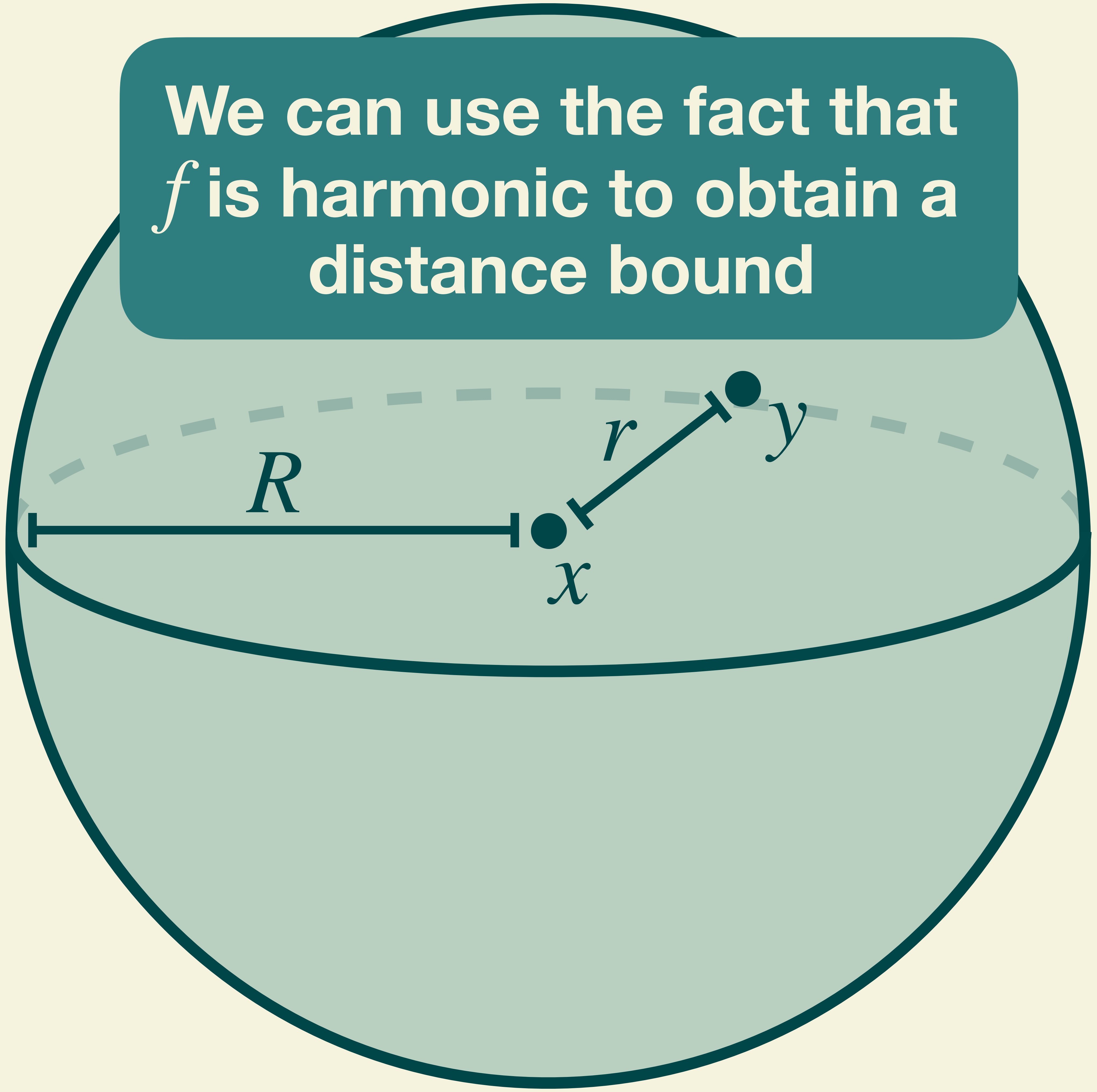
$$\frac{1 - r/R}{(1 + r/R)^2} f(x) \leq f(y) \leq \frac{1 + r/R}{(1 - r/R)^2} f(x)$$

lower bound — upper bound

always safe to take step of size

$$\frac{R}{2} \left| a + 2 - \sqrt{a^2 + 8a} \right|,$$

where $a = \dfrac{f(x)}{f^*}$



Main idea: get distance bounds from Harnack's inequality

Let f be a positive harmonic function on a ball:

What if f is not positive?

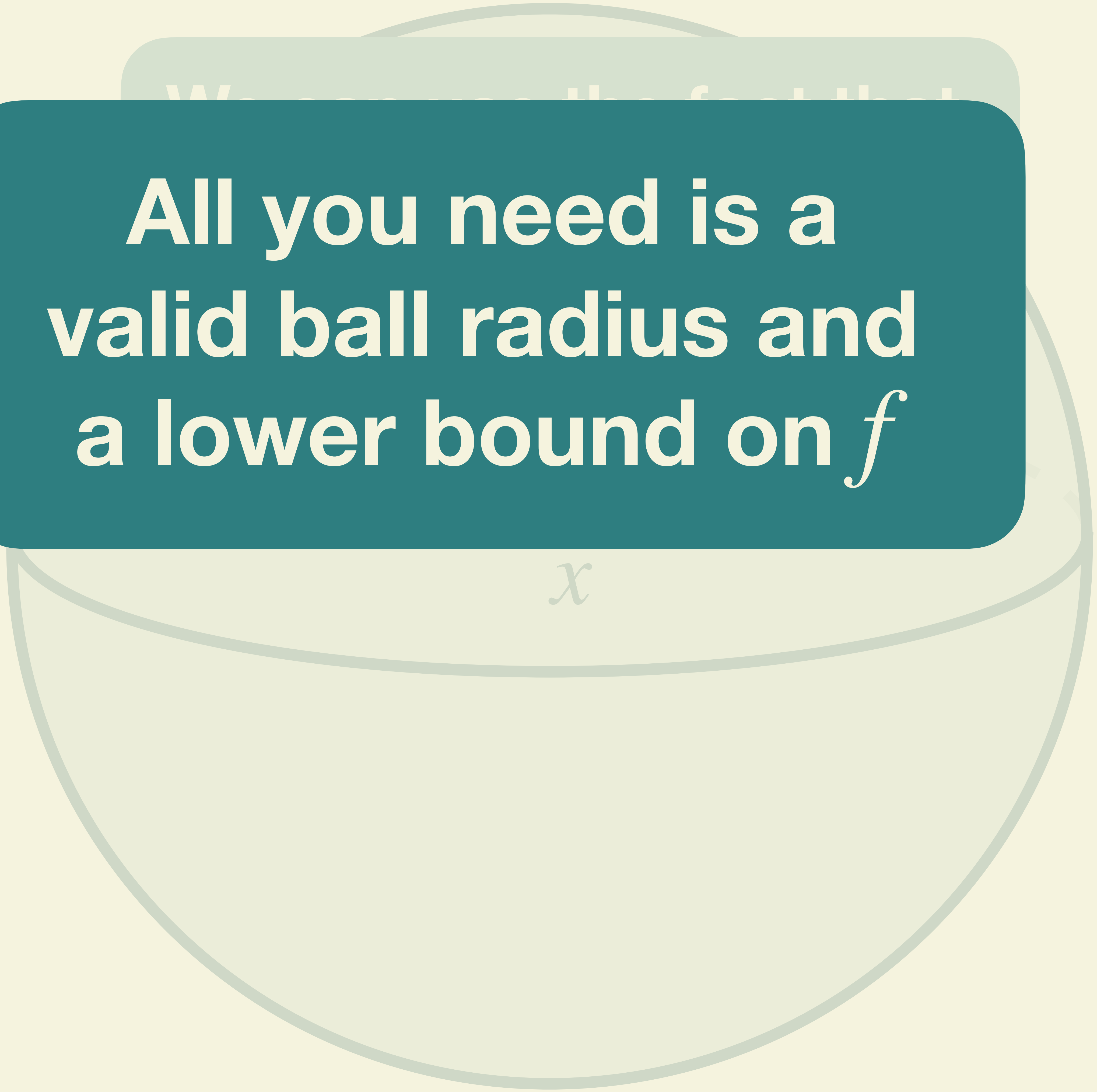
Just add a constant to make it positive on the ball

upper bound

always safe to take step of size

$$r = \frac{R}{2}\left|a + 2 - \sqrt{a^2 + 8a}\right|,$$

where $a = \frac{f(x)}{f^*}$



All you need is a valid ball radius and a lower bound on f

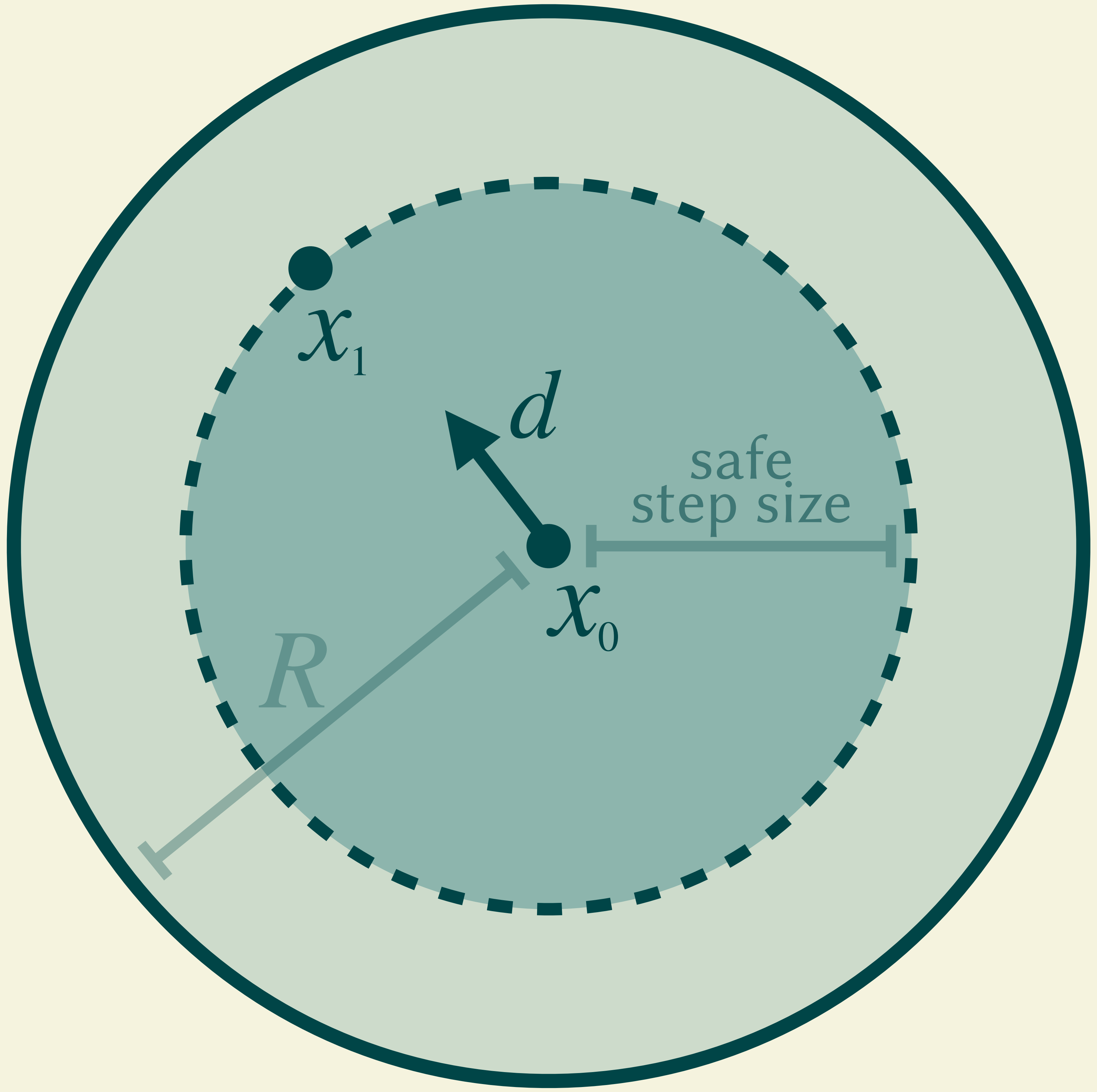
Algorithm sketch

Harnack Tracing

Starting from point x_0 in direction d:

- Pick ball radius
- Shift f to be positive on ball
- Calculate safe step size
- Take safe step in ray direction

Repeat until f is sufficiently close to f^*



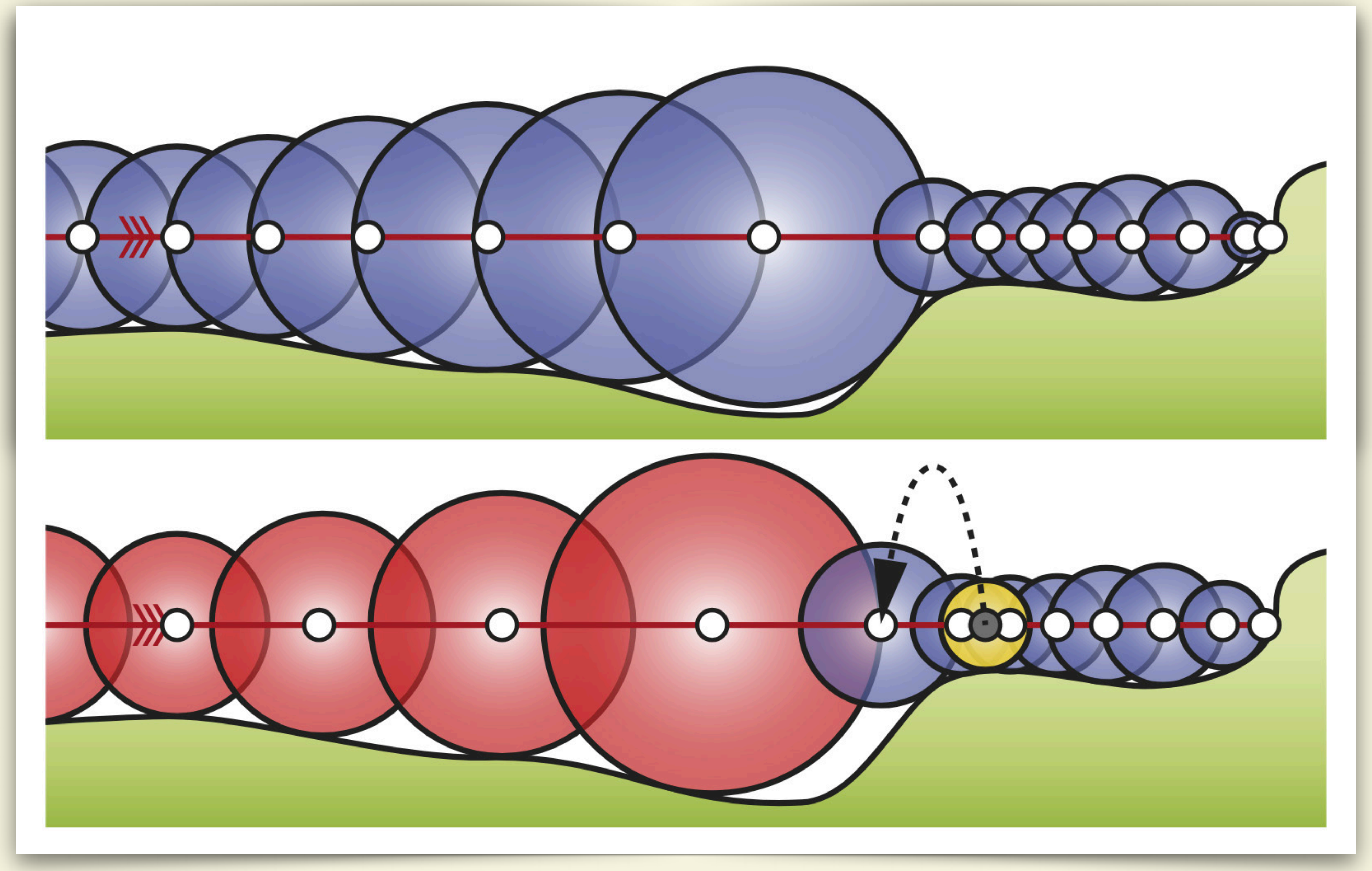
2D example

Sphere tracing acceleration

[Keinert *et al.* 2014]: “over-stepping”

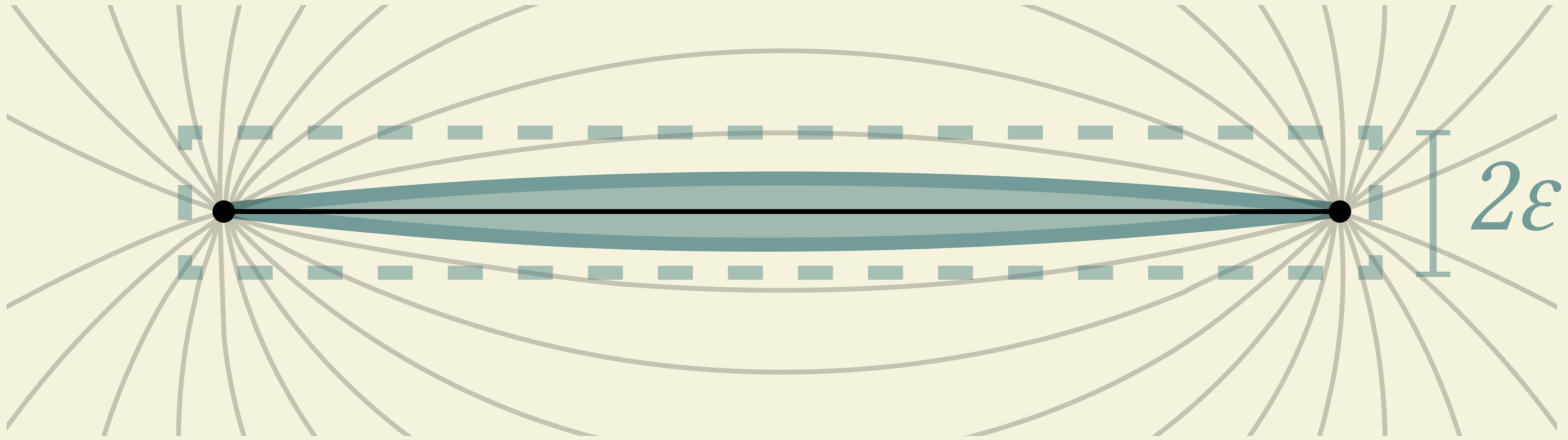
conservative steps

valid oversteps

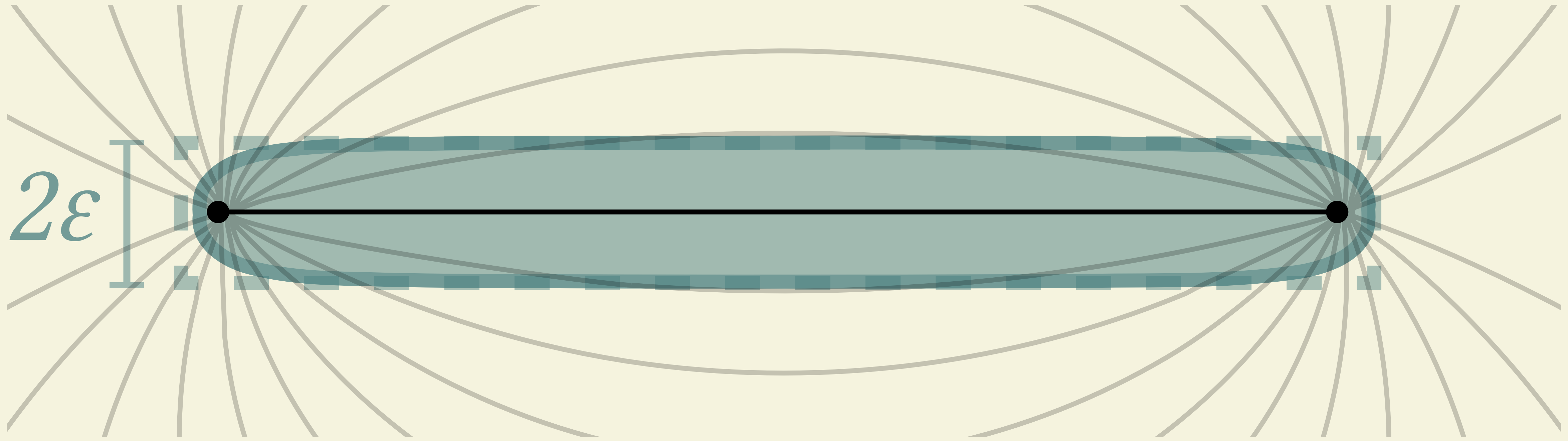


Acceleration: gradient termination

How do you decide when you have “hit” the surface?



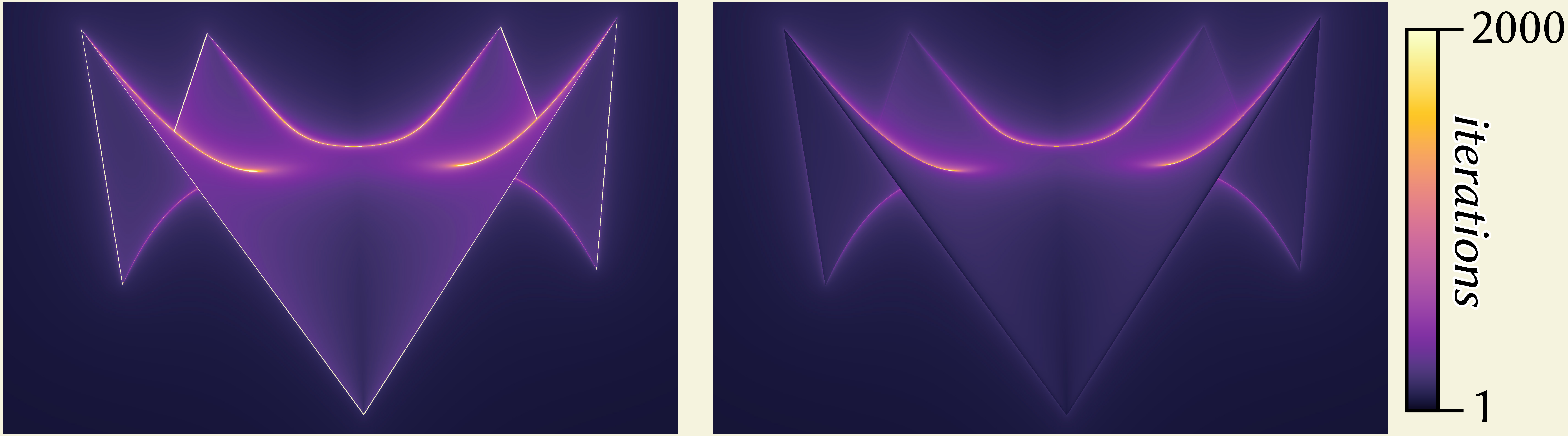
$$|f(\mathbf{x}) - f^*| < \varepsilon$$



$$\frac{|f(\mathbf{x}) - f^*|}{|\nabla f(\mathbf{x})|} < \varepsilon$$

Acceleration: gradient termination

How do you decide when you have “hit” the surface?

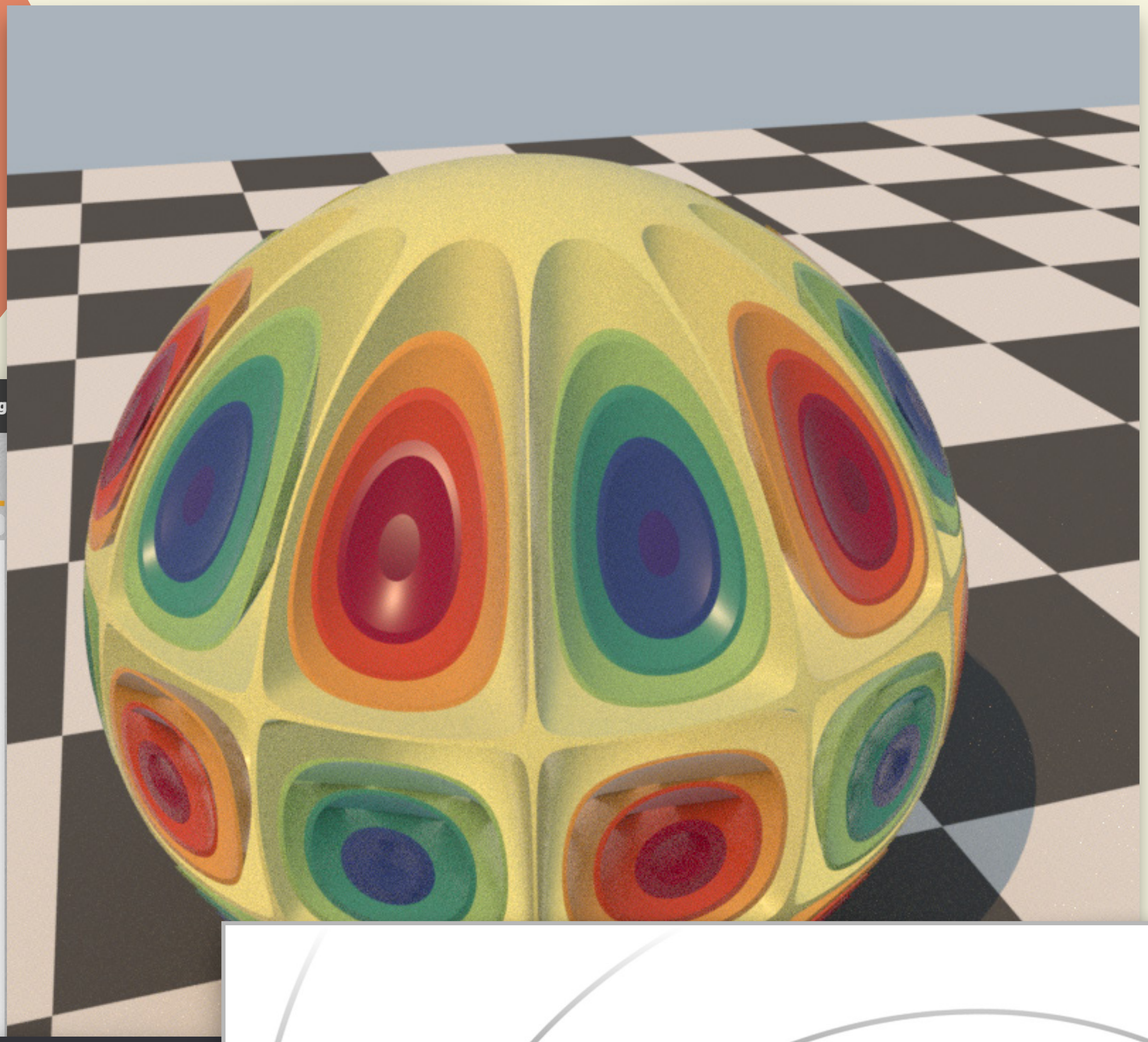


$$|f(\mathbf{x}) - f^*| < \varepsilon$$

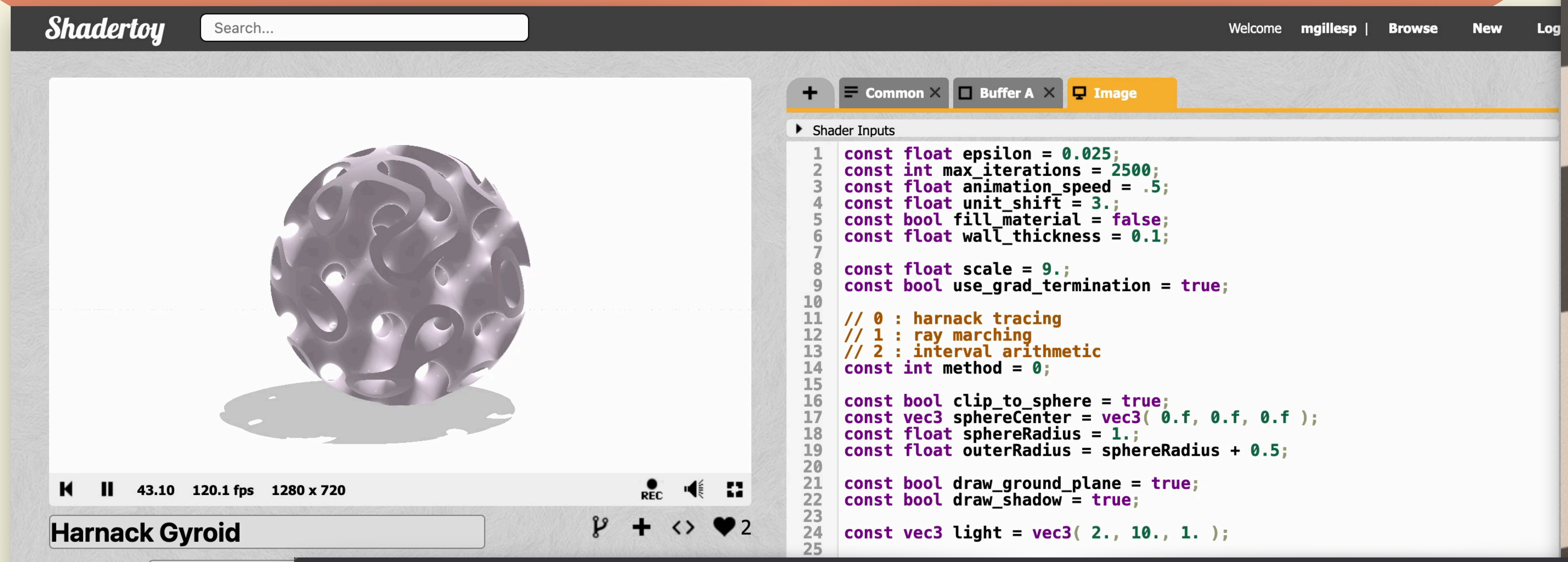
$$\frac{|f(\mathbf{x}) - f^*|}{|\nabla f(\mathbf{x})|} < \varepsilon$$

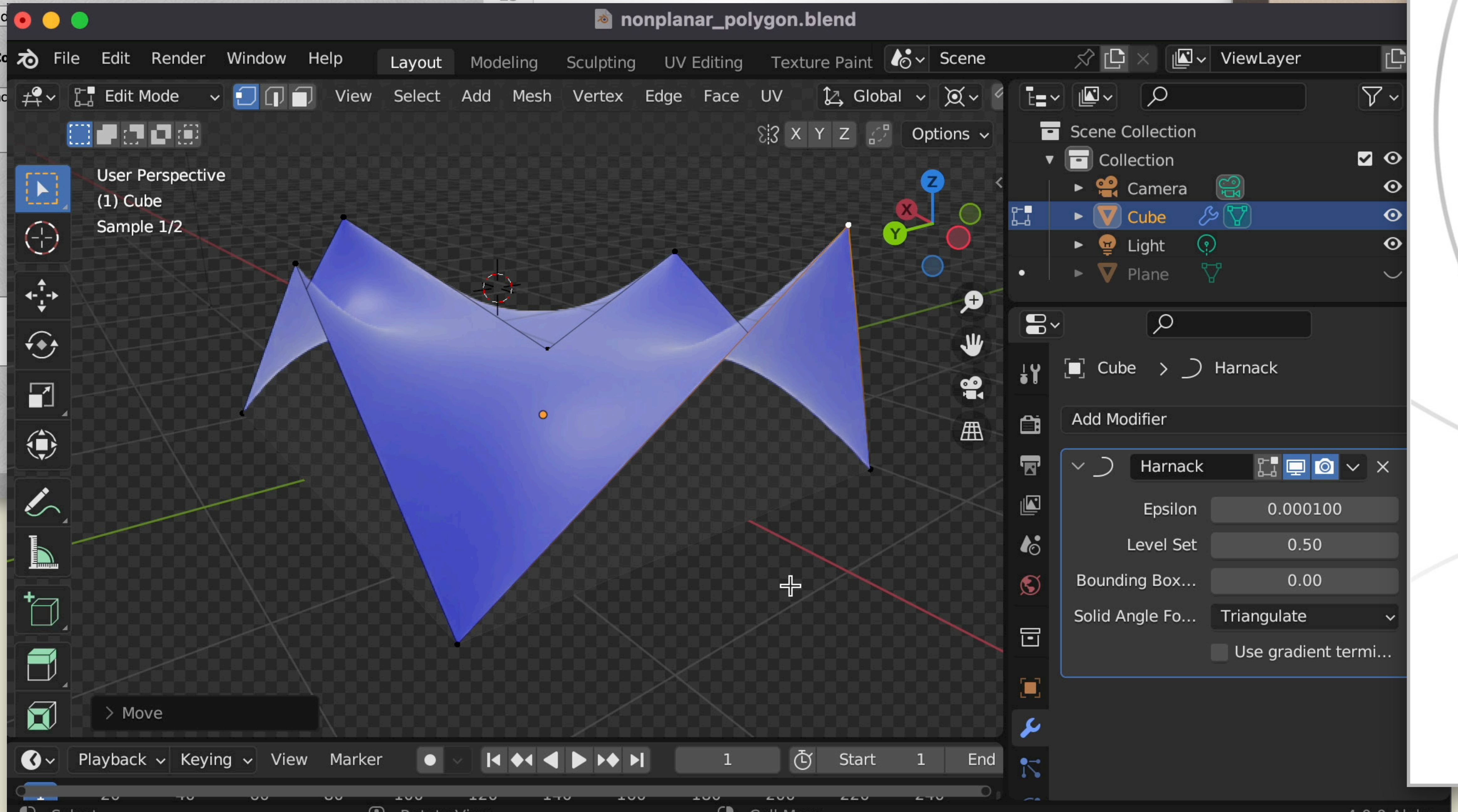
Simple to implement

PBRT (CPU ray tracer)

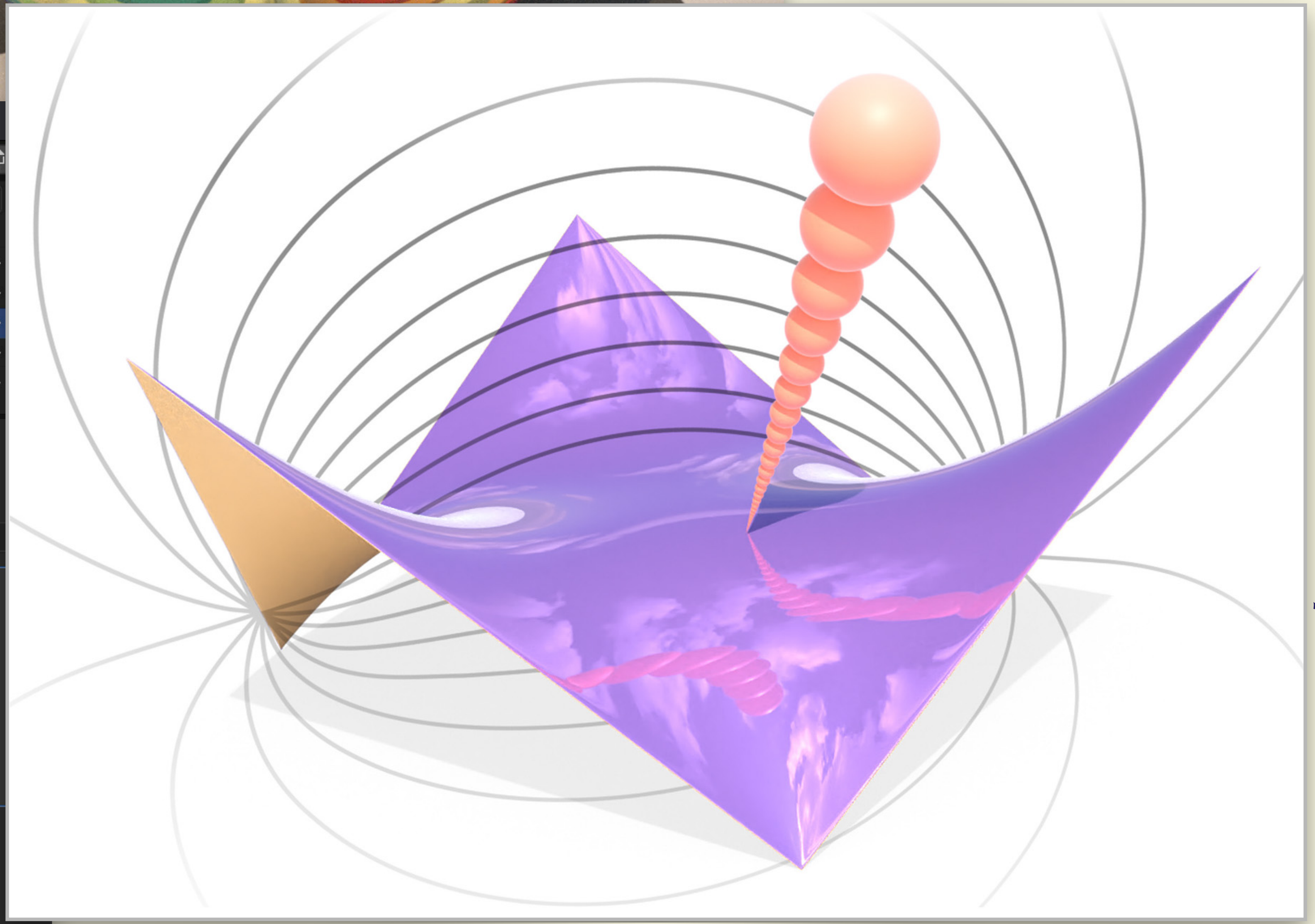


ShaderToy (WebGL shaders)



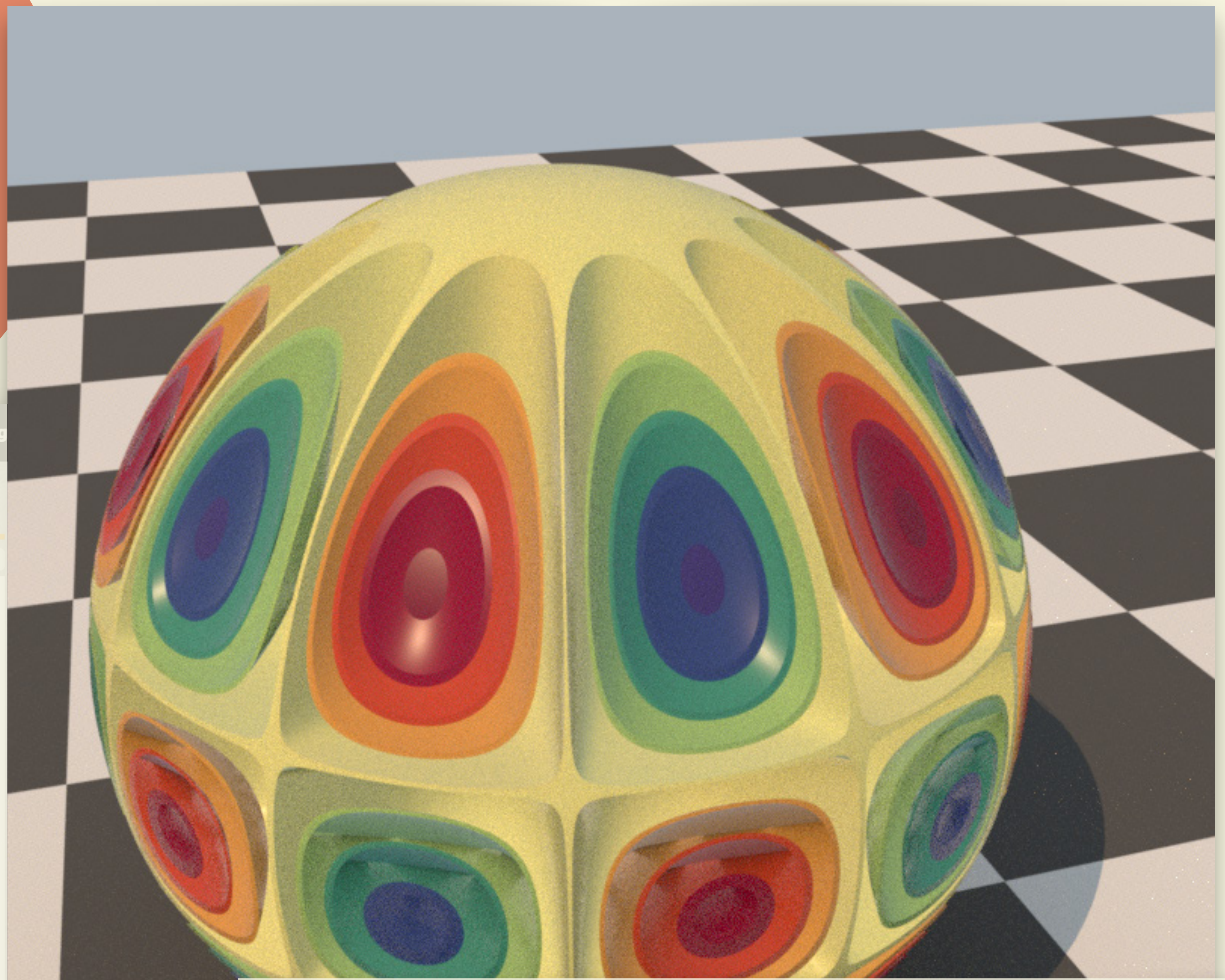


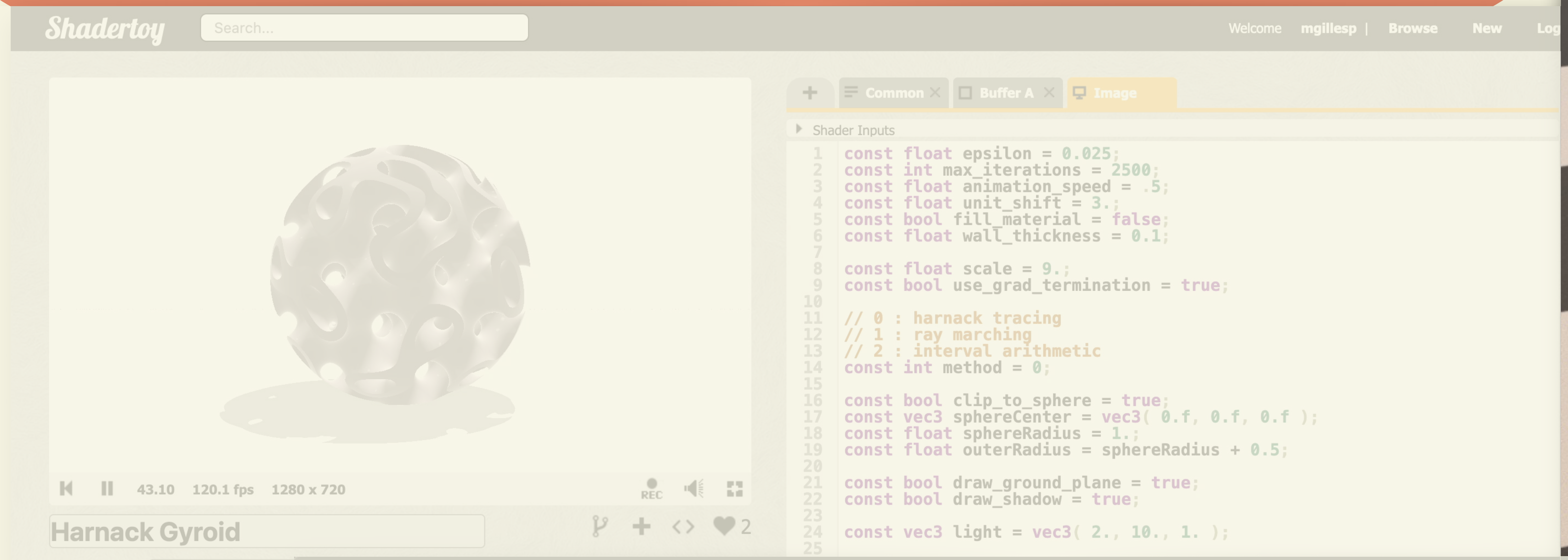
Blender (CPU ray tracer)



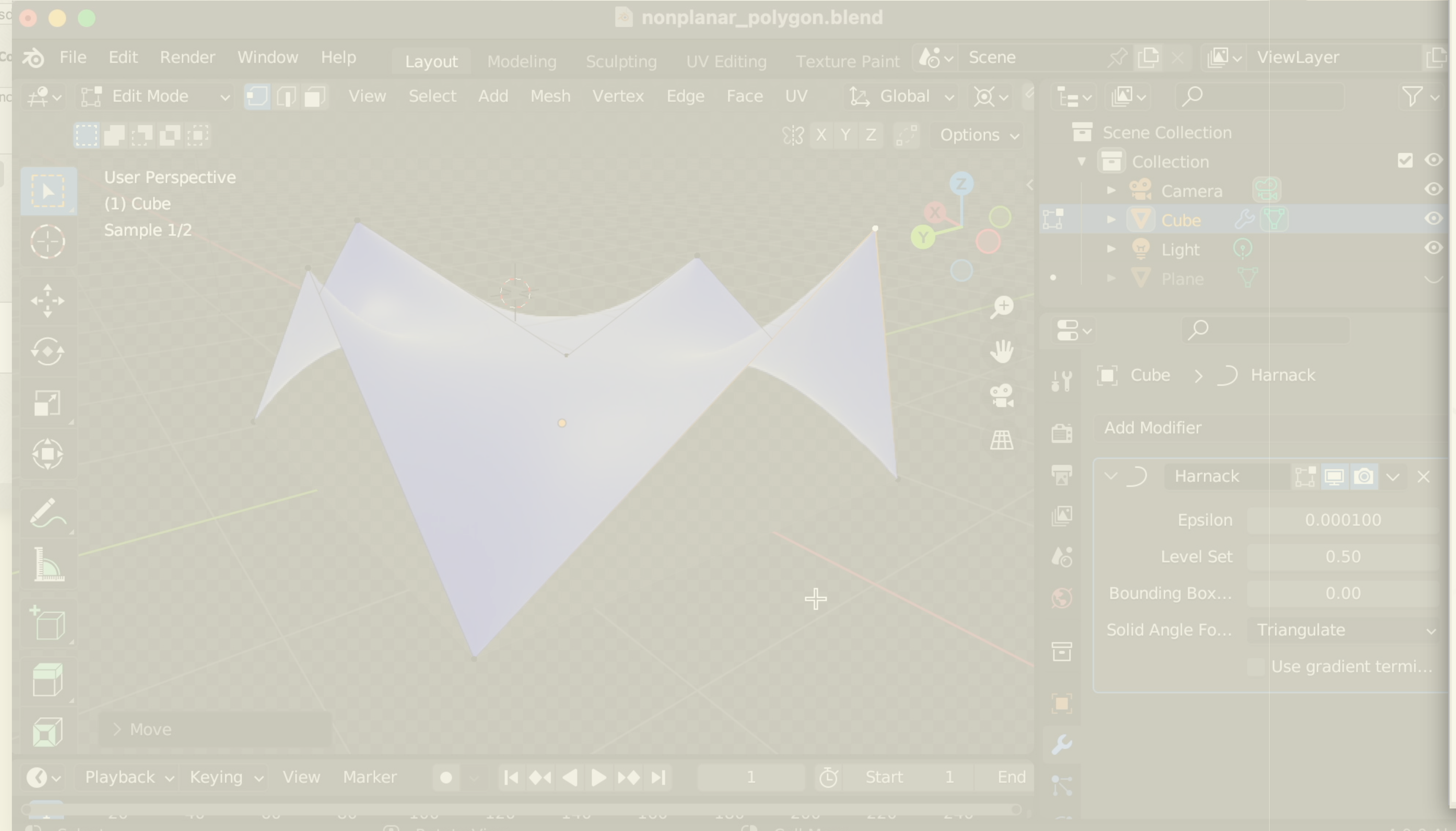
Simple to implement

PBRT (CPU ray tracer)





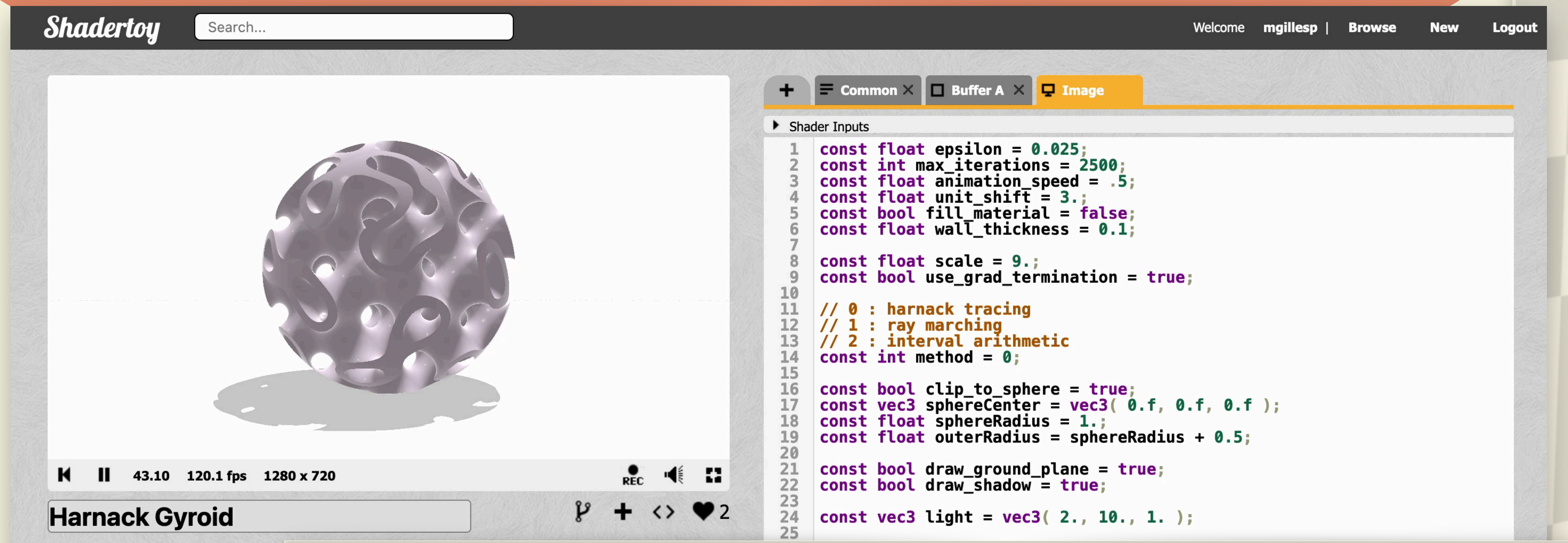
ShaderToy (WebGL shaders)

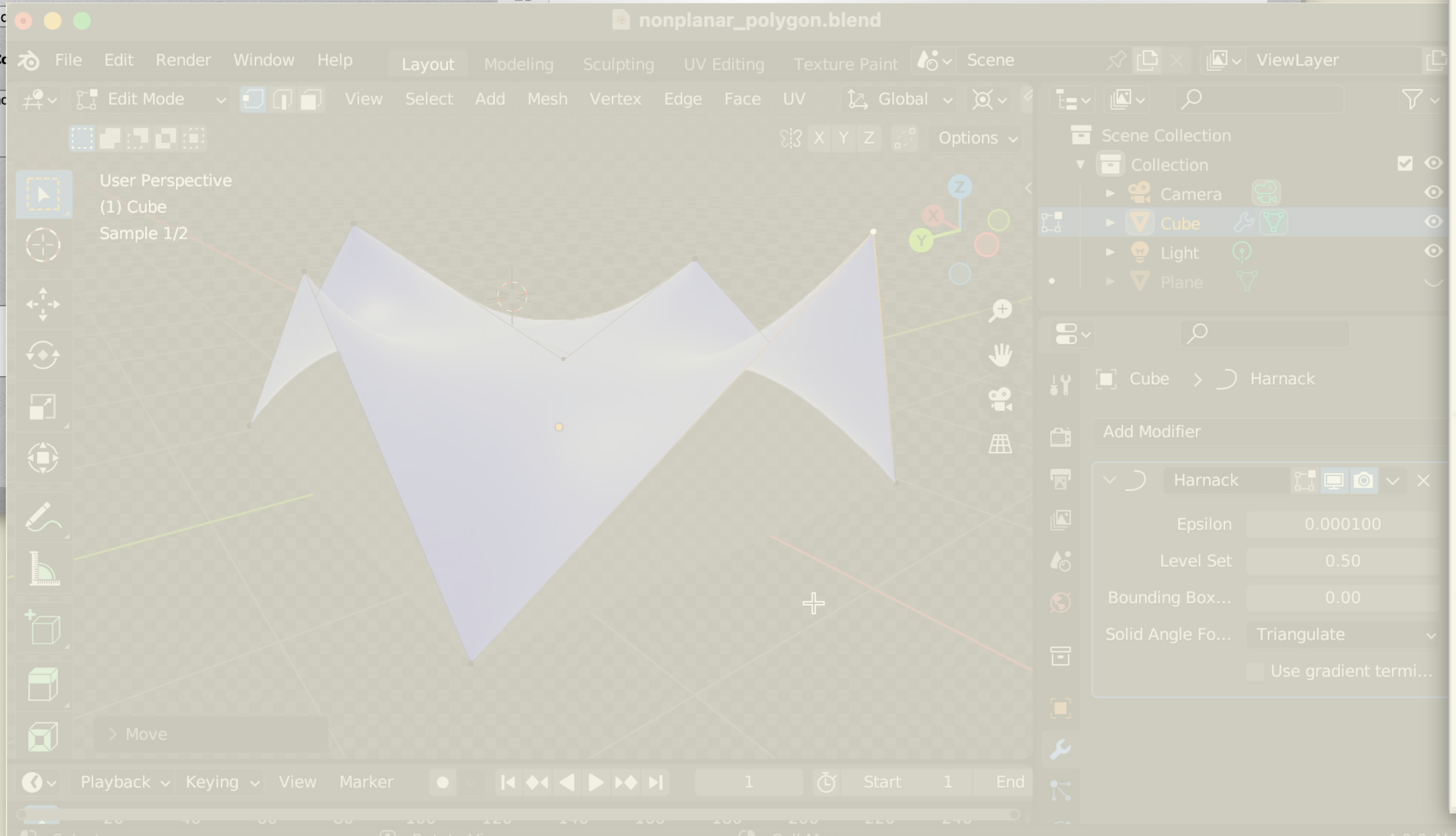


Blender (CPU ray tracer)

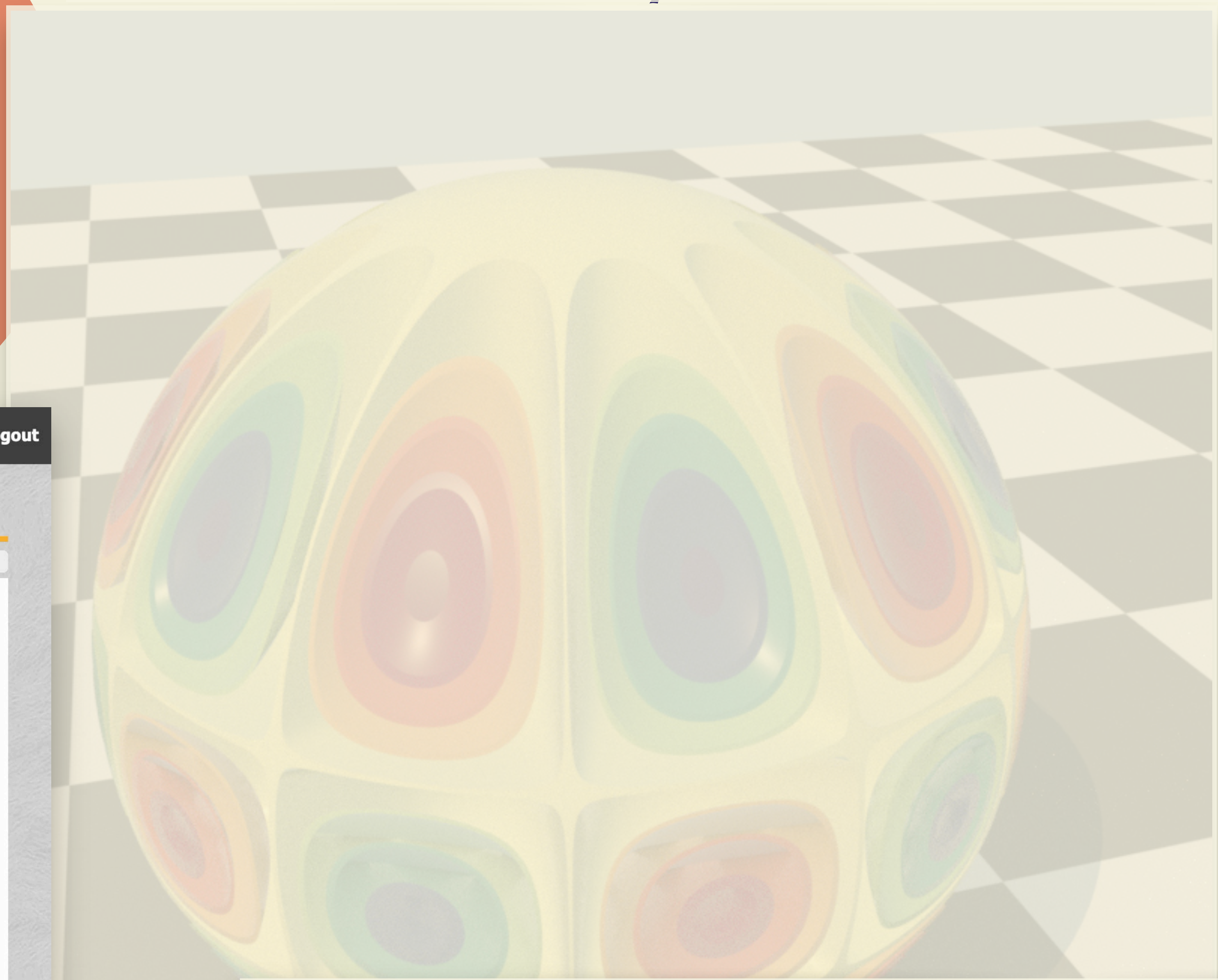
Simple to implement

ShaderToy (WebGL shaders)





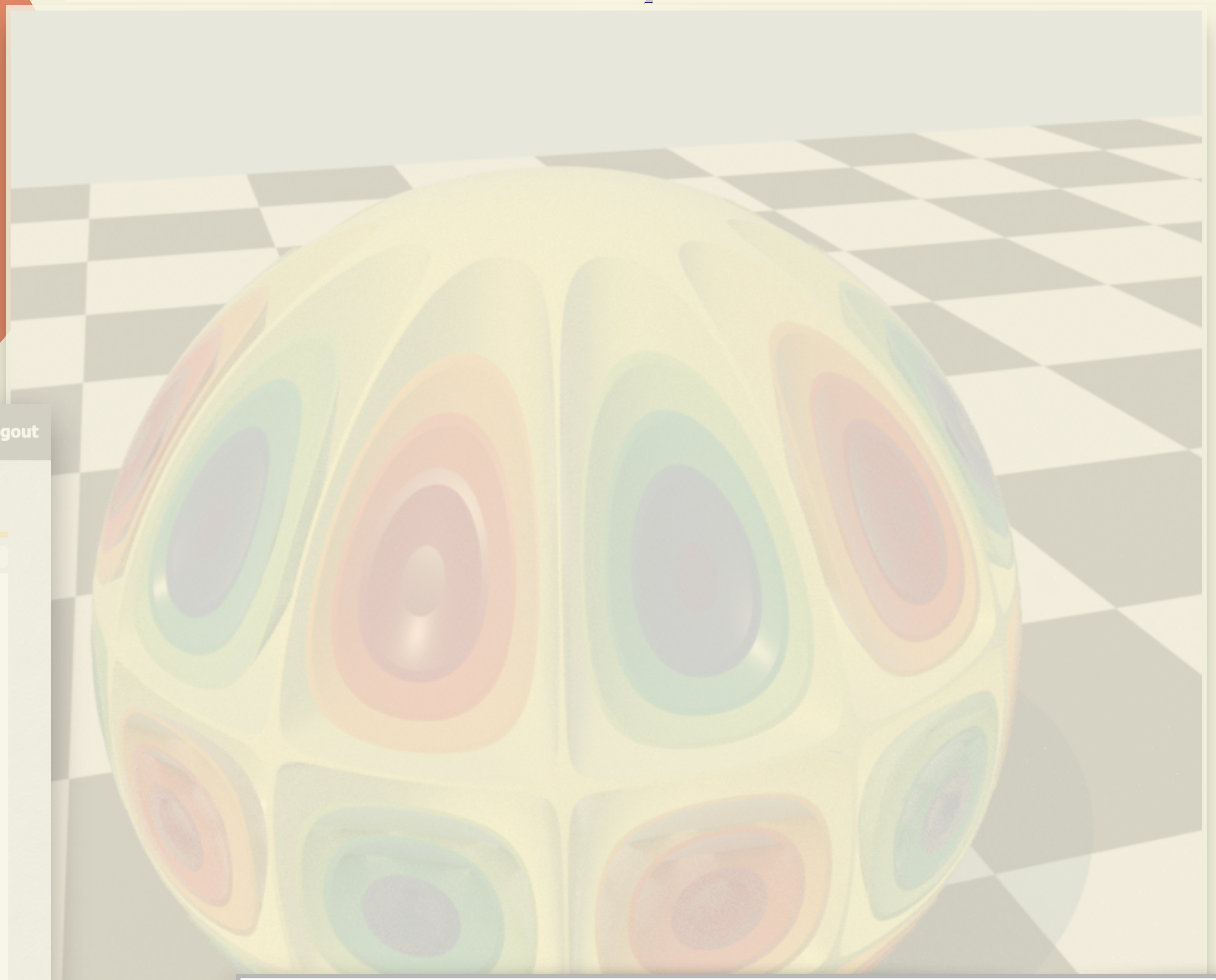
PBRT (CPU ray tracer)



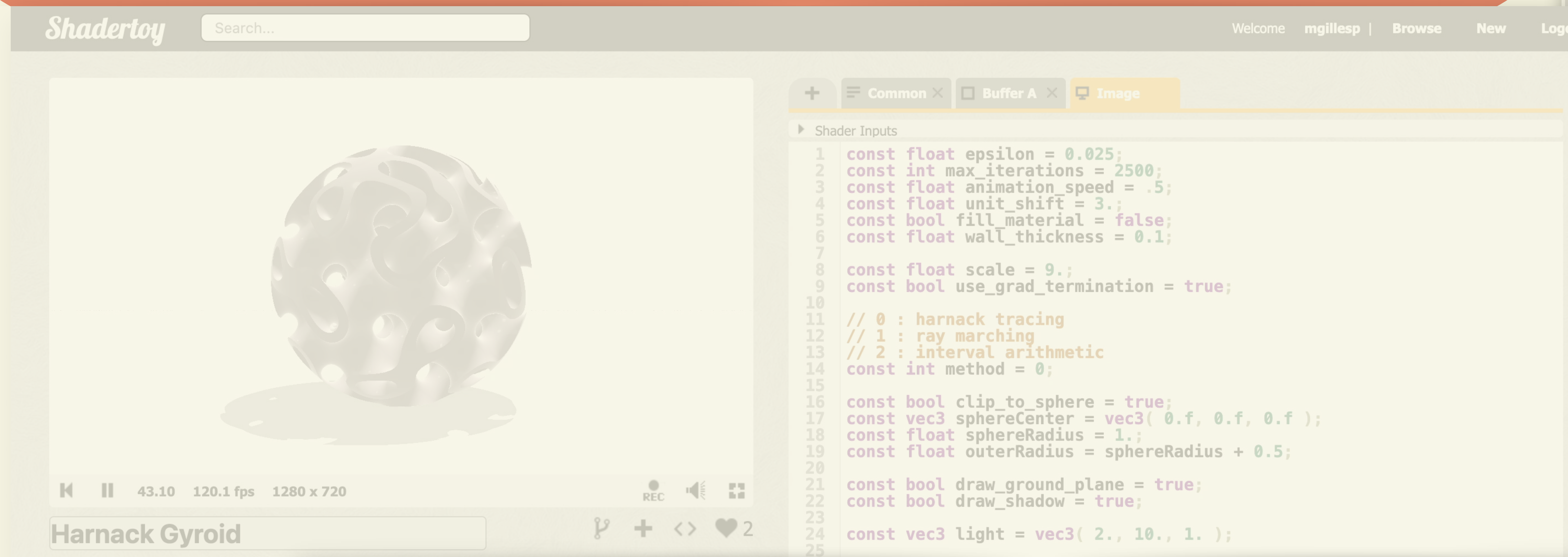
Blender (CPU ray tracer)

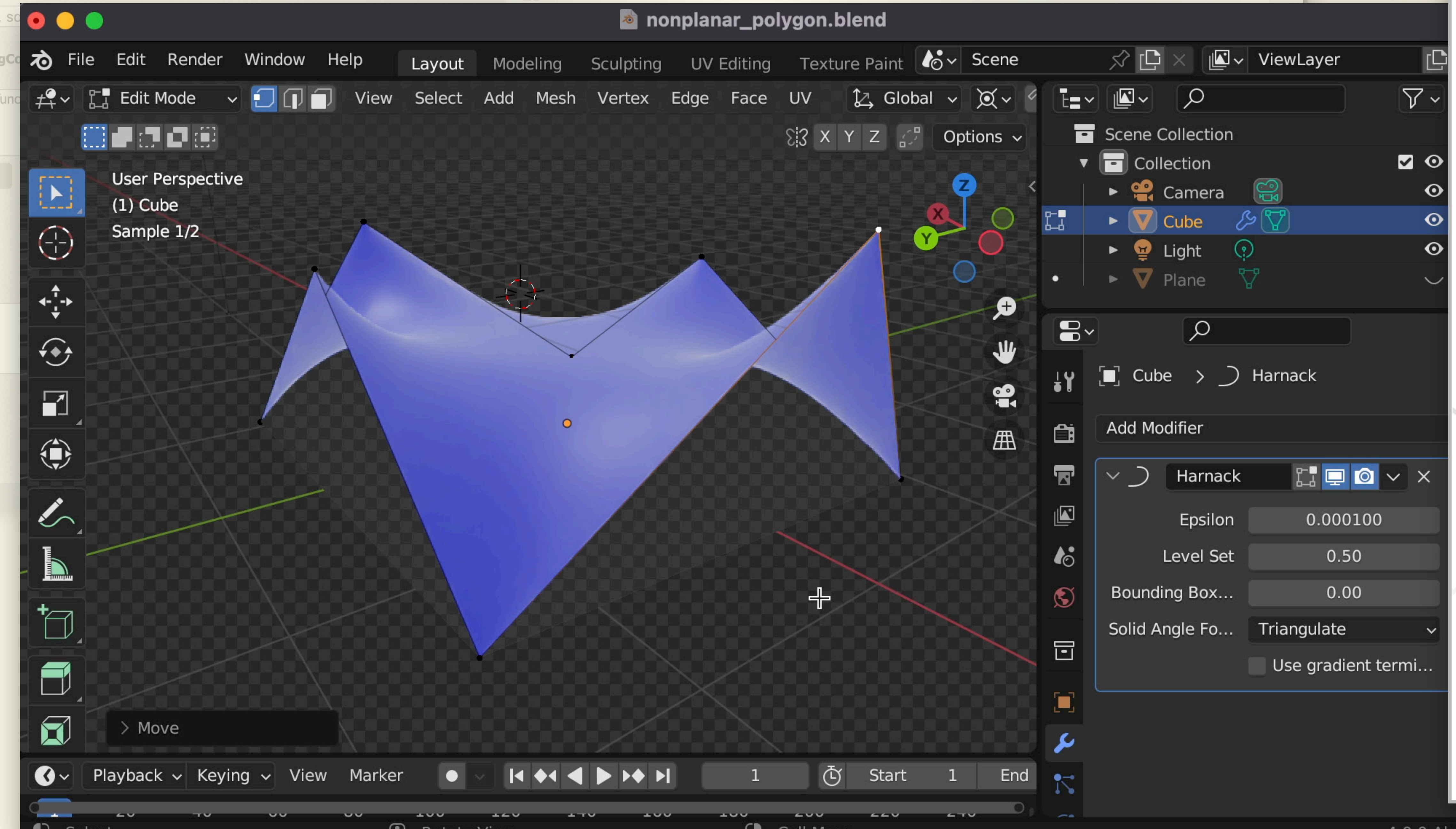
Simple to implement

PBRT (CPU ray tracer)

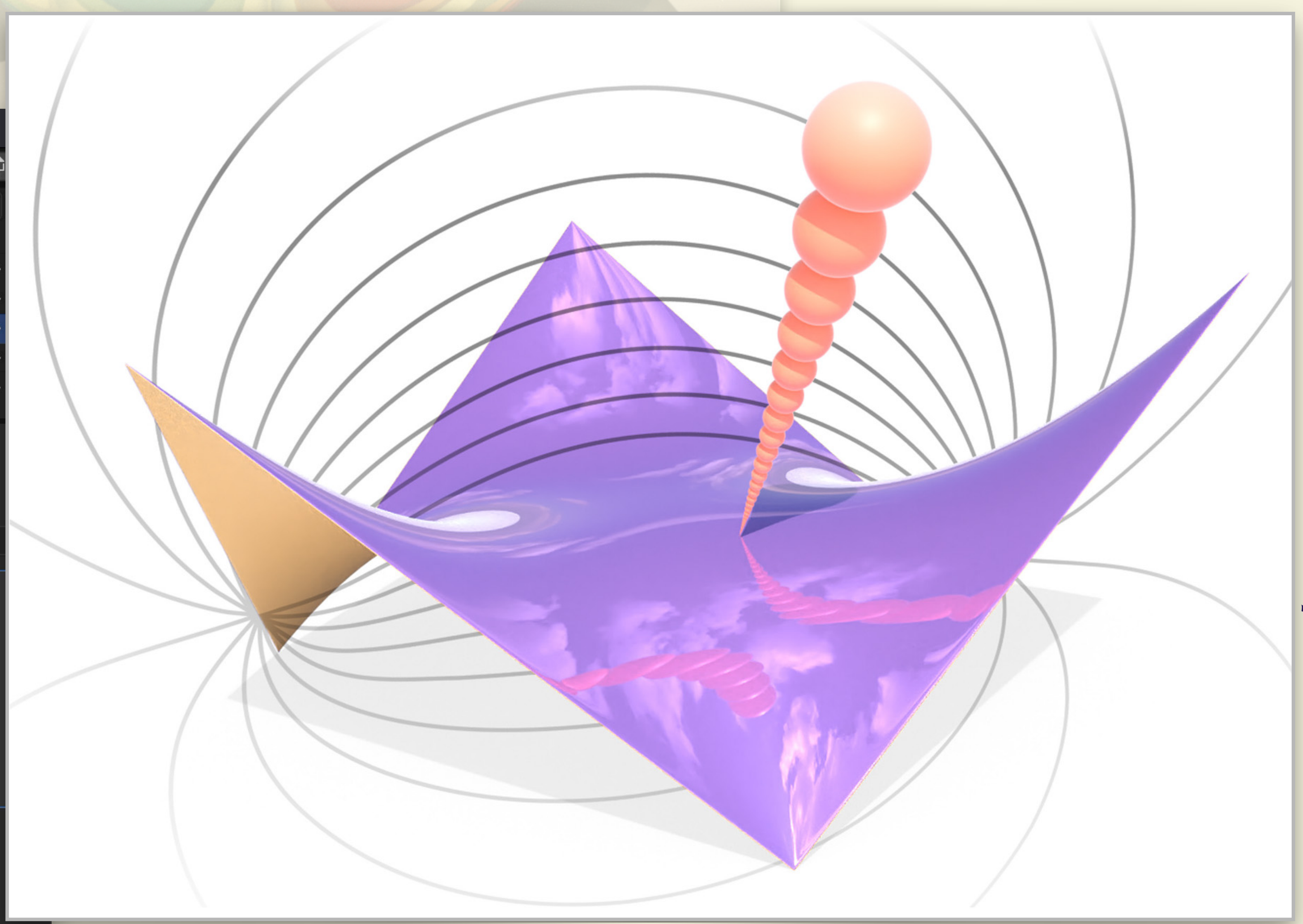


ShaderToy (WebGL shaders)





Blender (CPU ray tracer)

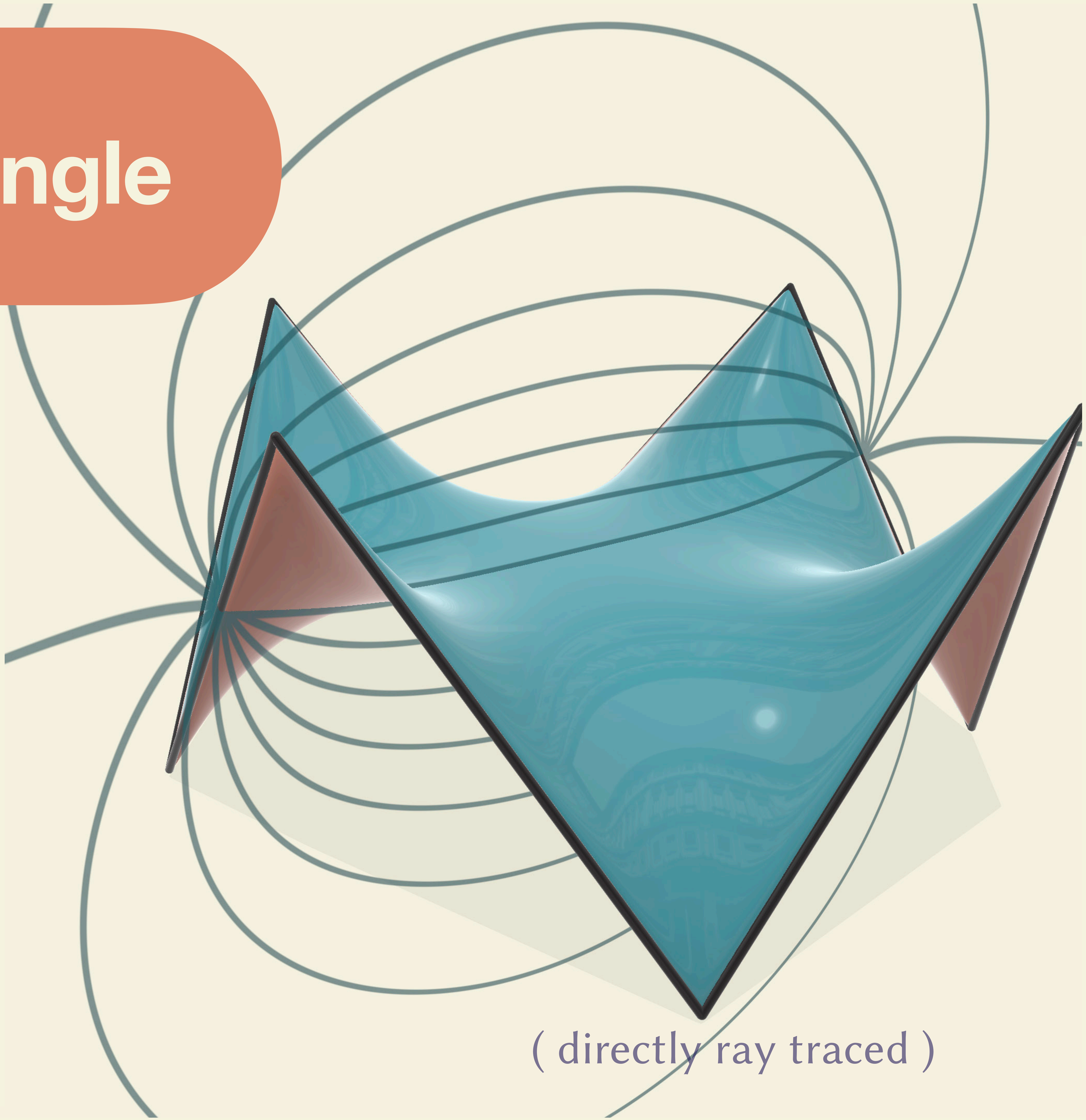


Generalized winding number

[Jacobson et al. 2013]

a.k.a. *signed solid angle*

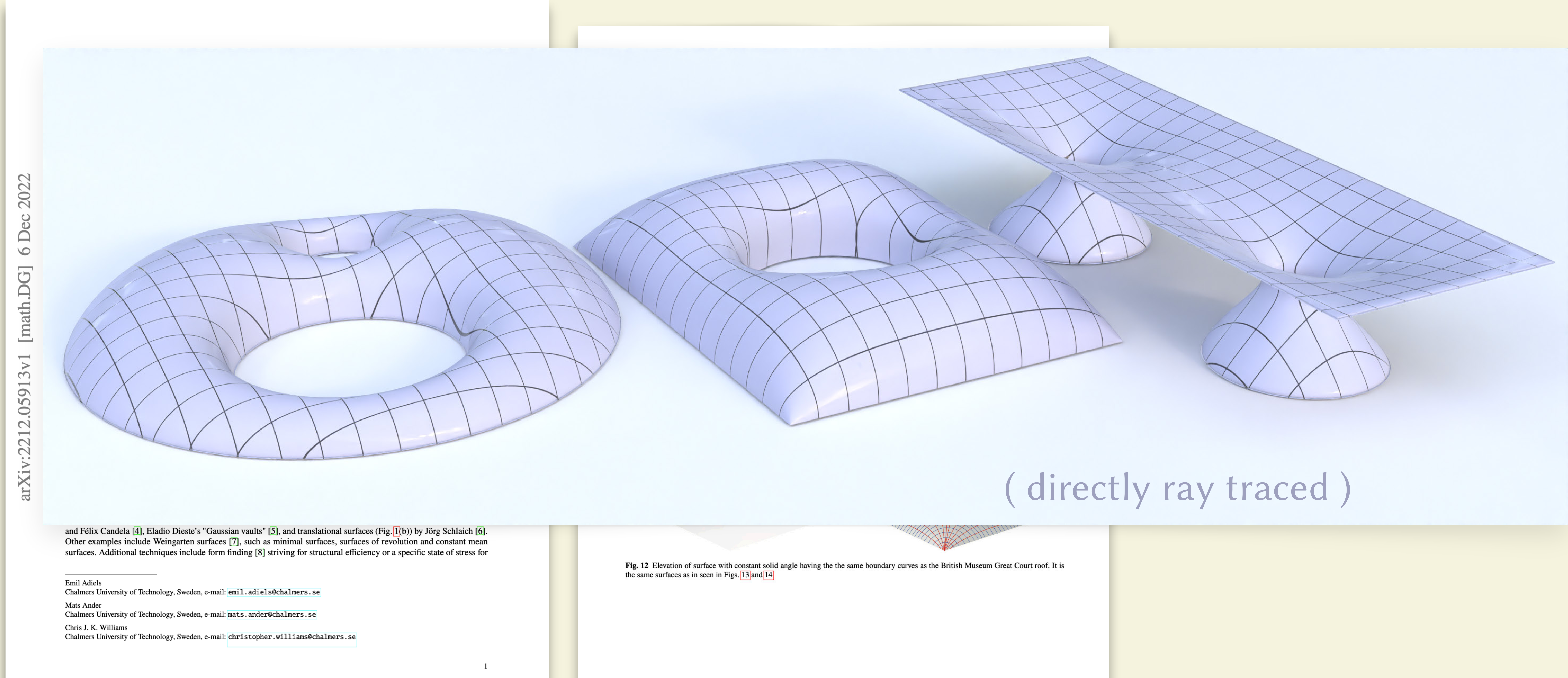
Signed solid angle



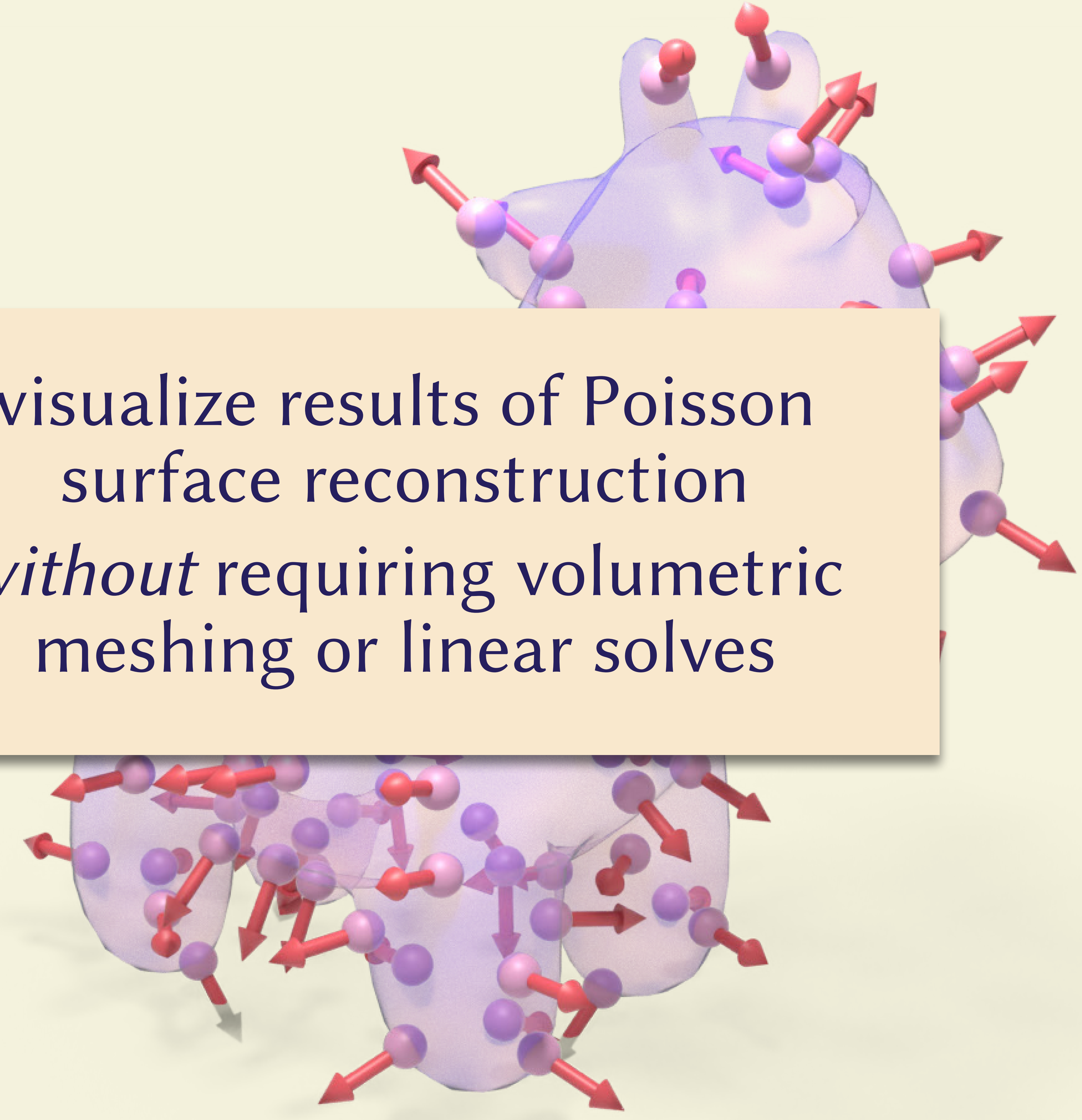
(directly ray traced)

Architectural grid shells

[Adiels et al. 2022]



Surface reconstruction
[Kazhdan et al. 2006]



visualize results of Poisson surface reconstruction *without* requiring volumetric meshing or linear solves

(directly ray traced)

Riemann surfaces

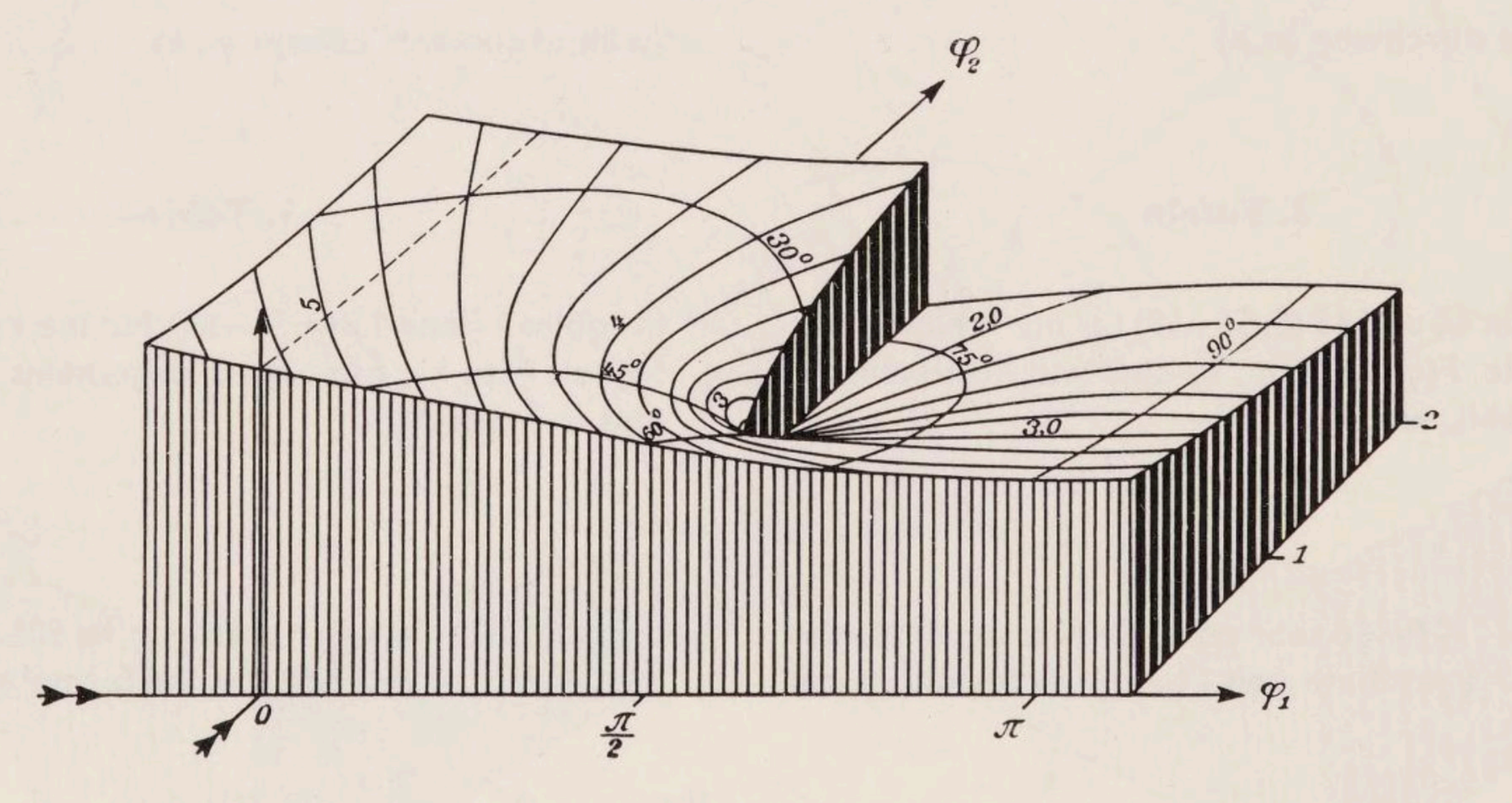
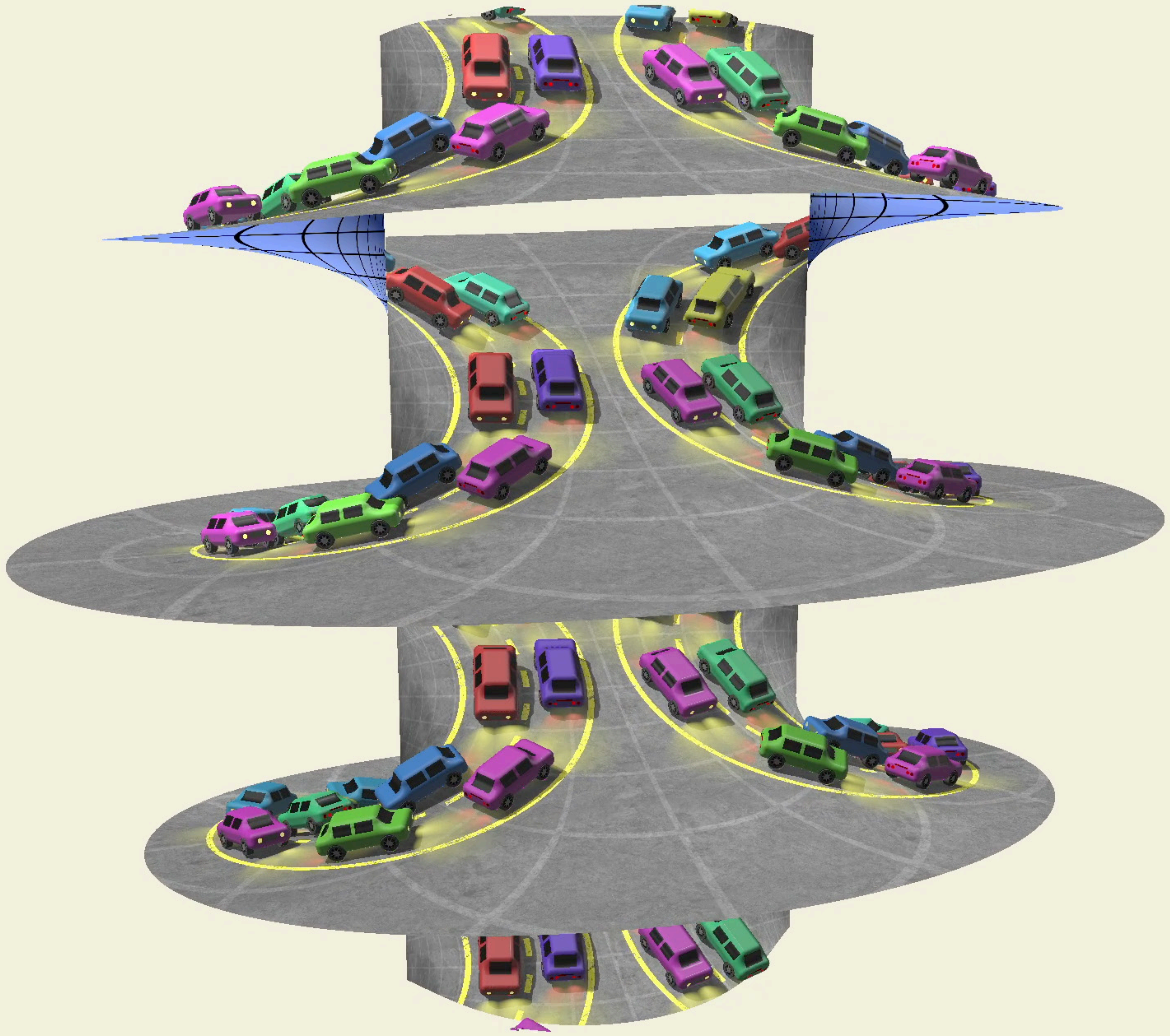


Fig. 24 Relief des 2. Zweiges der Funktion $F(\varphi, k)$ mit $k = 0{,}8$. $(\varphi = \varphi_1 + i\varphi_2)$

Fig. 24 Relief of the 2 nd branch of the function $F(\varphi, k)$ with $k = 0{,}8$. $(\varphi = \varphi_1 + i\varphi_2)$

[Jahnke, Emde & Lösch 1960]



(directly ray traced)

*technically the trigonometric approximation to the gyroid

The gyroid

[Diegel 2021]

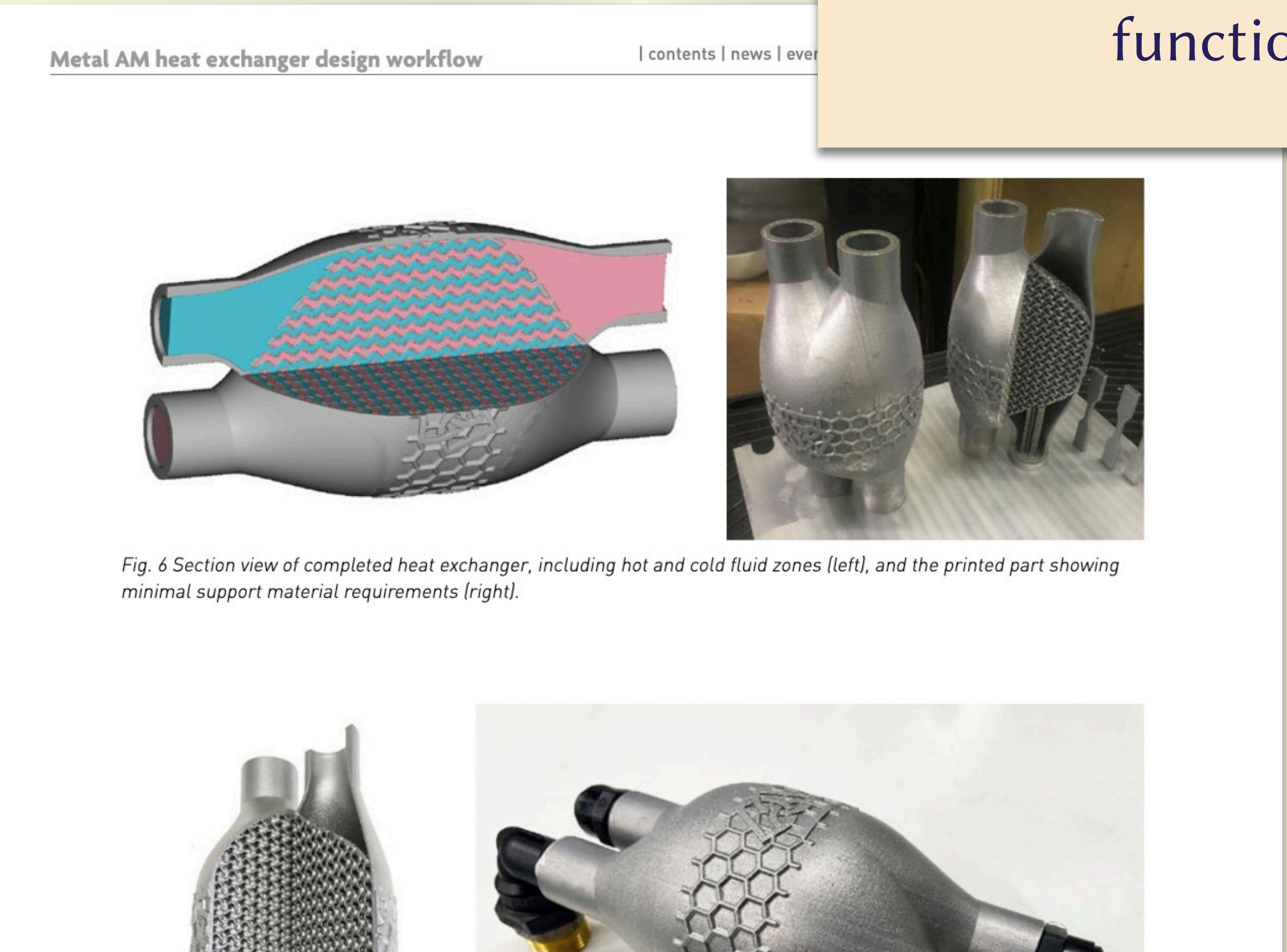
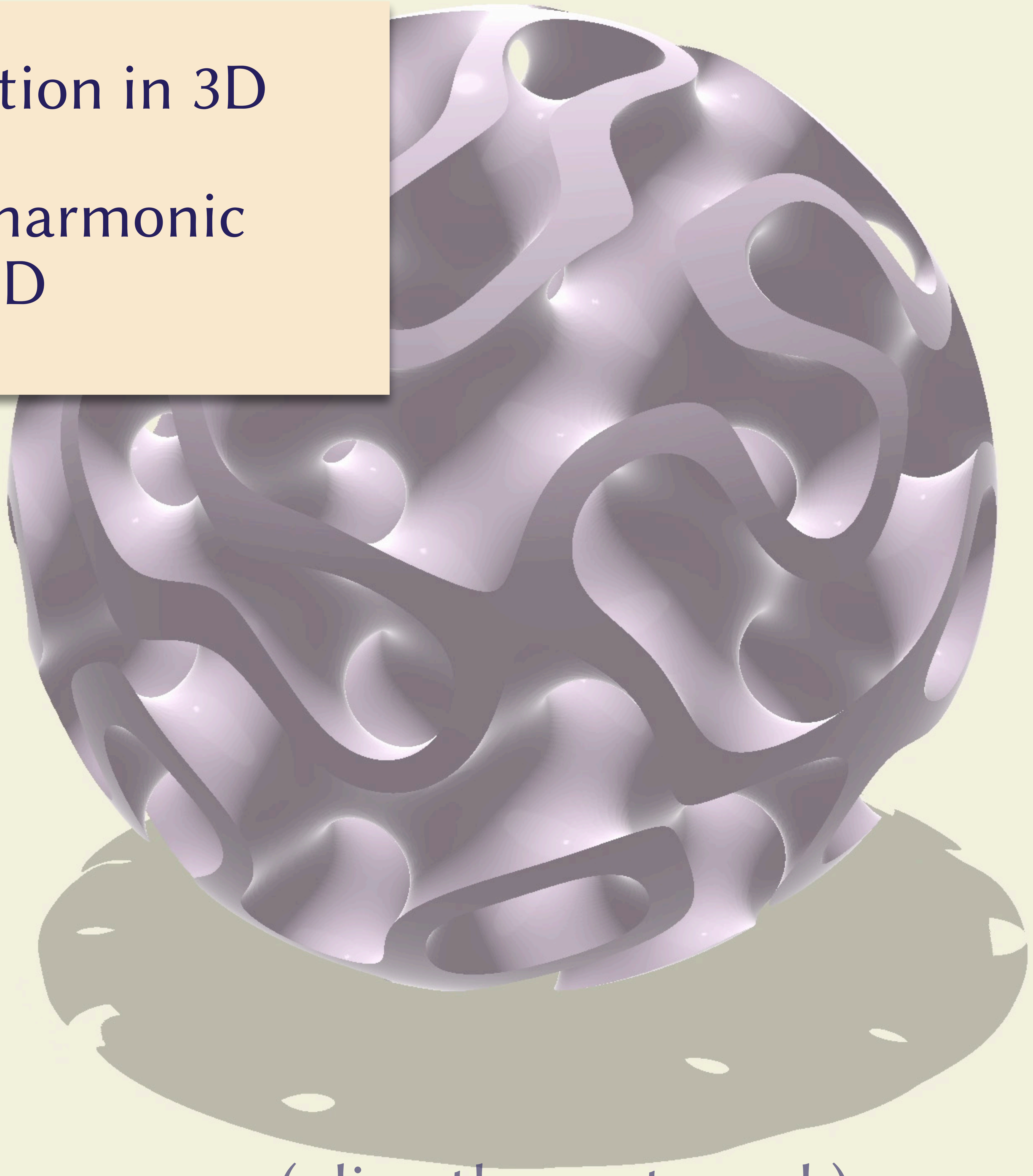


Fig. 6 Section view of completed heat exchanger, including hot and cold fluid zones (left), and the printed part showing minimal support material requirements (right).

not a harmonic function in 3D

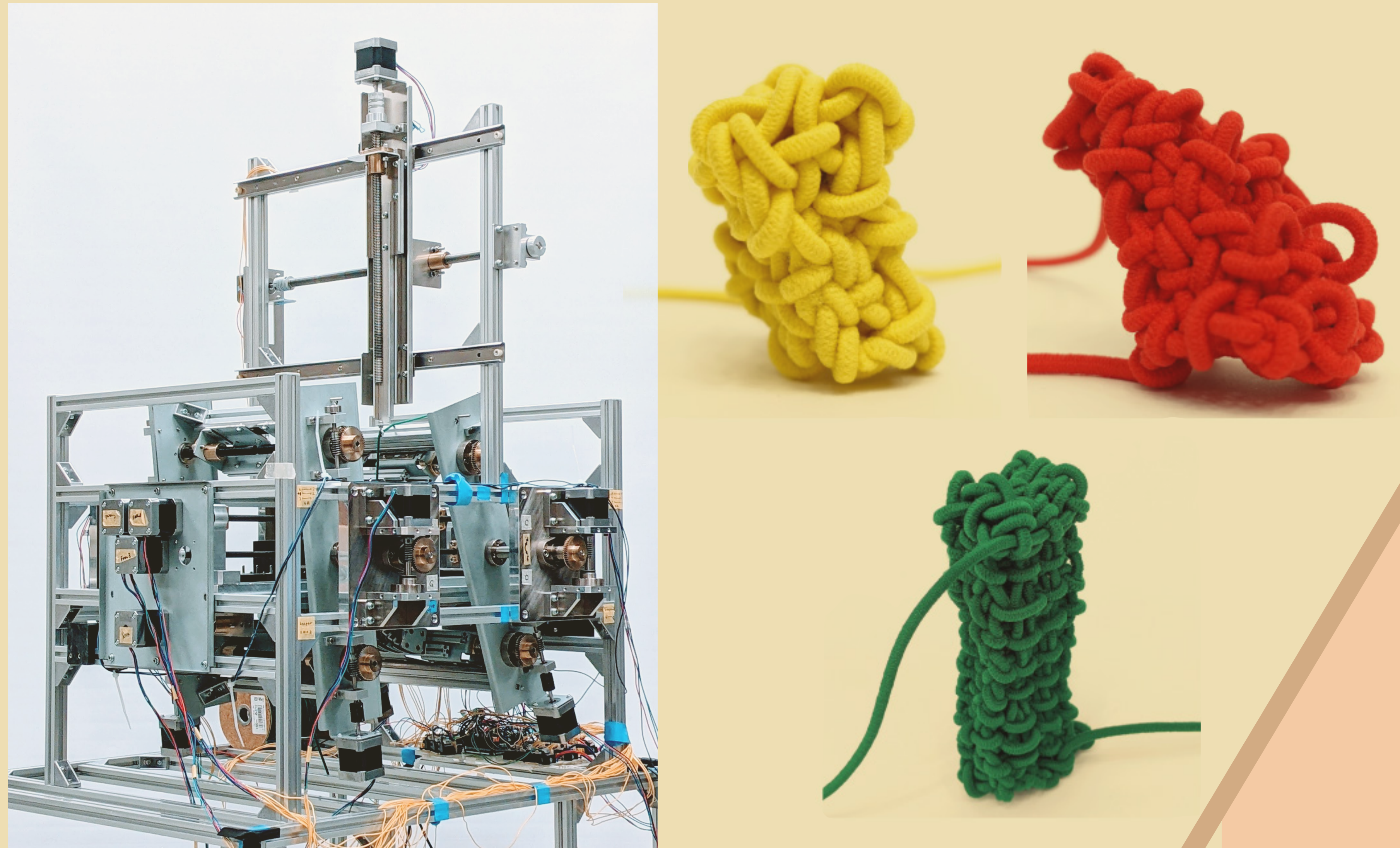
… but is a *slice* of a harmonic function in 4D



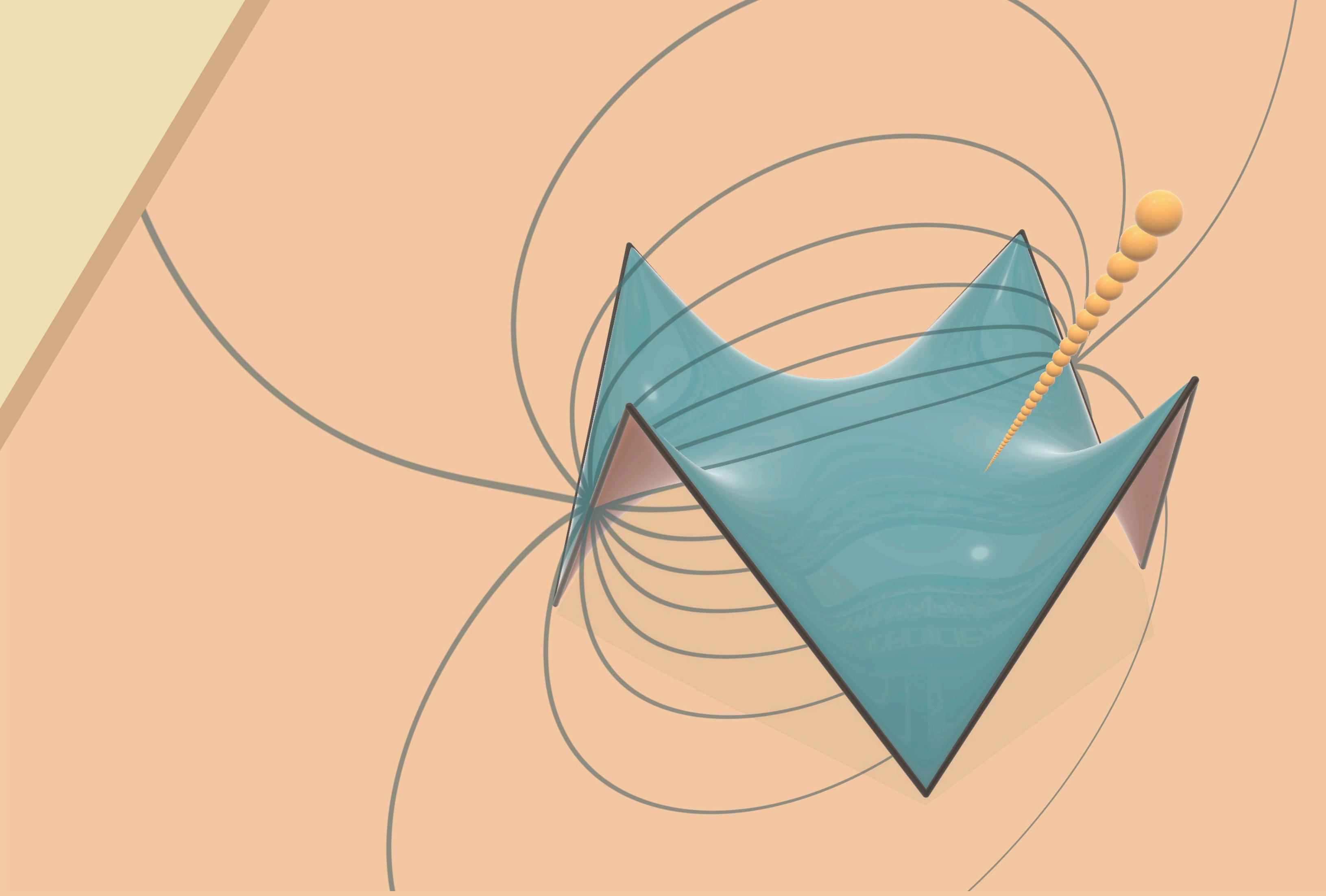
(directly ray traced)

More information about solid knitting (including instructions for solid knitting by hand) can be found at:

http://solid.knit.zone



Thanks!



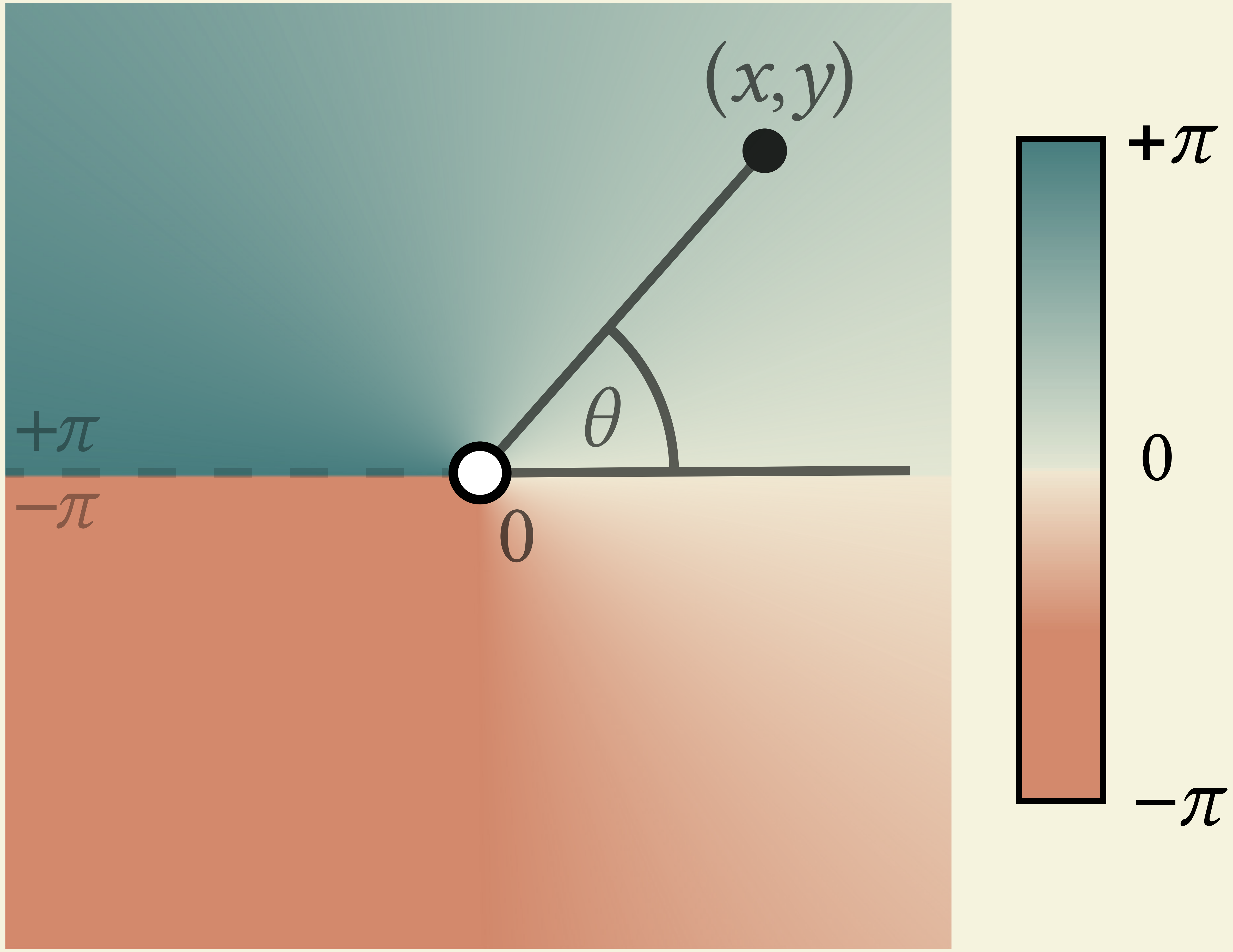
Links to Blender code and ShaderToy examples can be found at:

www.markjgillespie.com/Research/harnack-tracing

Angle-valued functions

$$\theta(x, y) = \mathrm{atan2}(x, y)$$

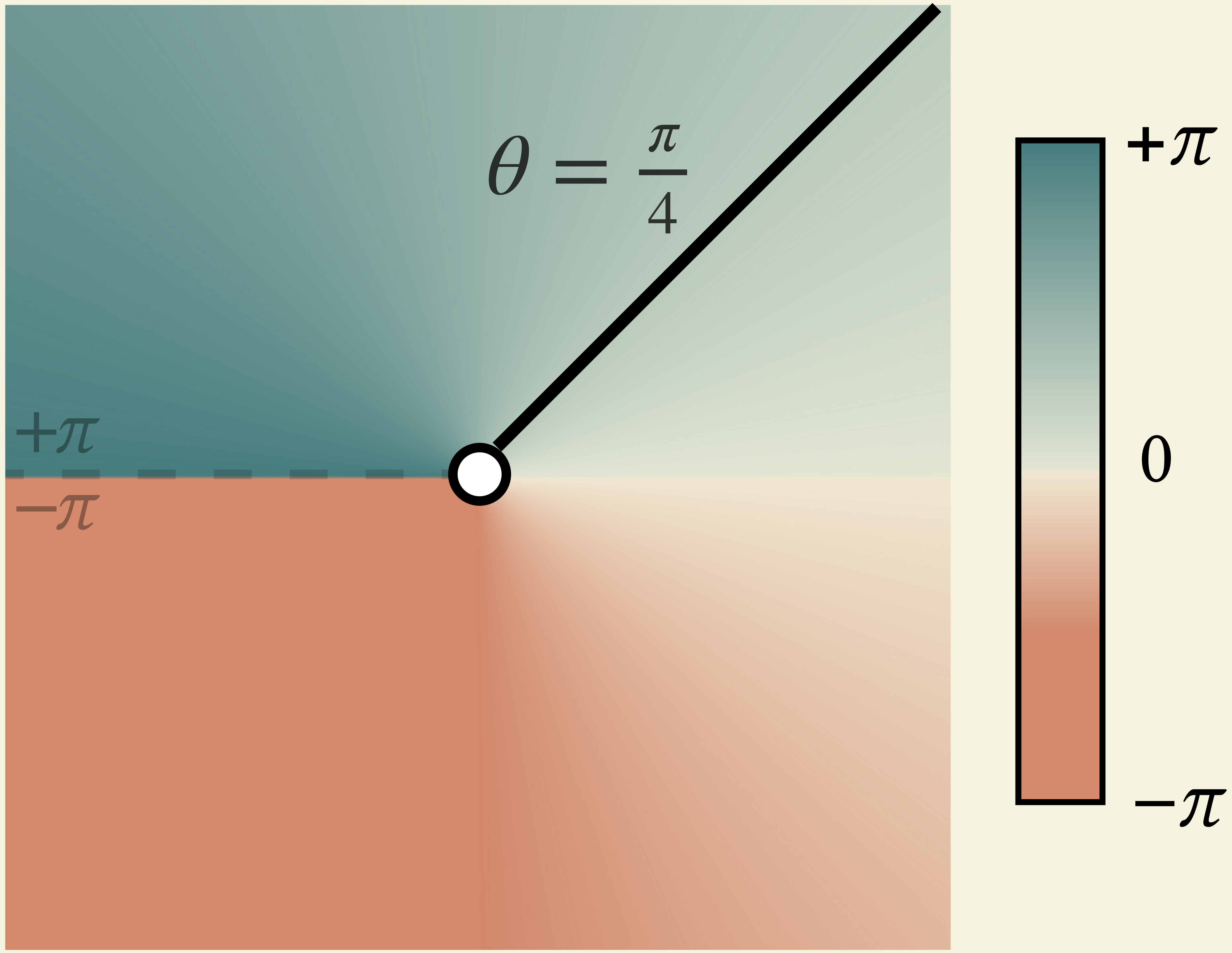
continuous when viewed modulo 2π



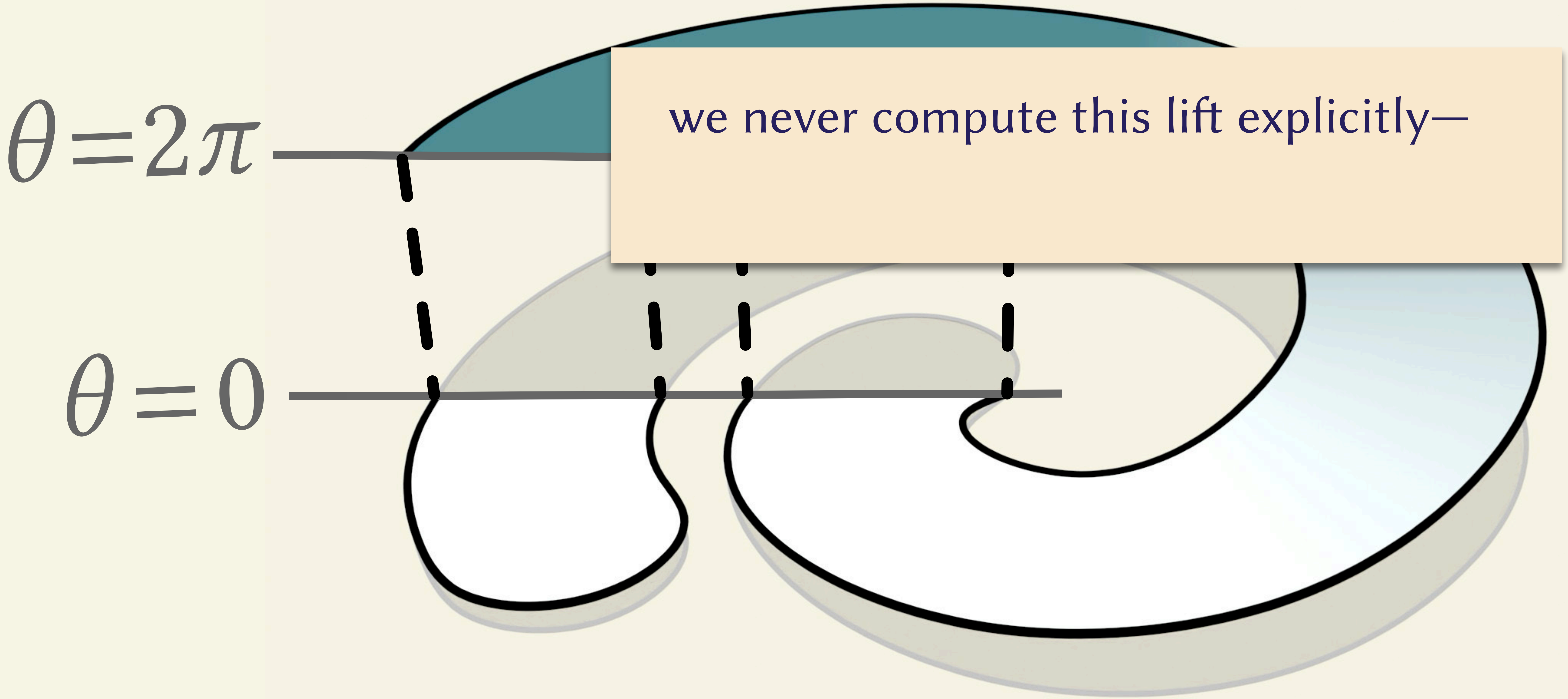
Angle-valued functions

continuous when viewed modulo 2π

$$\theta(x, y) = \text{atan2}(x, y)$$



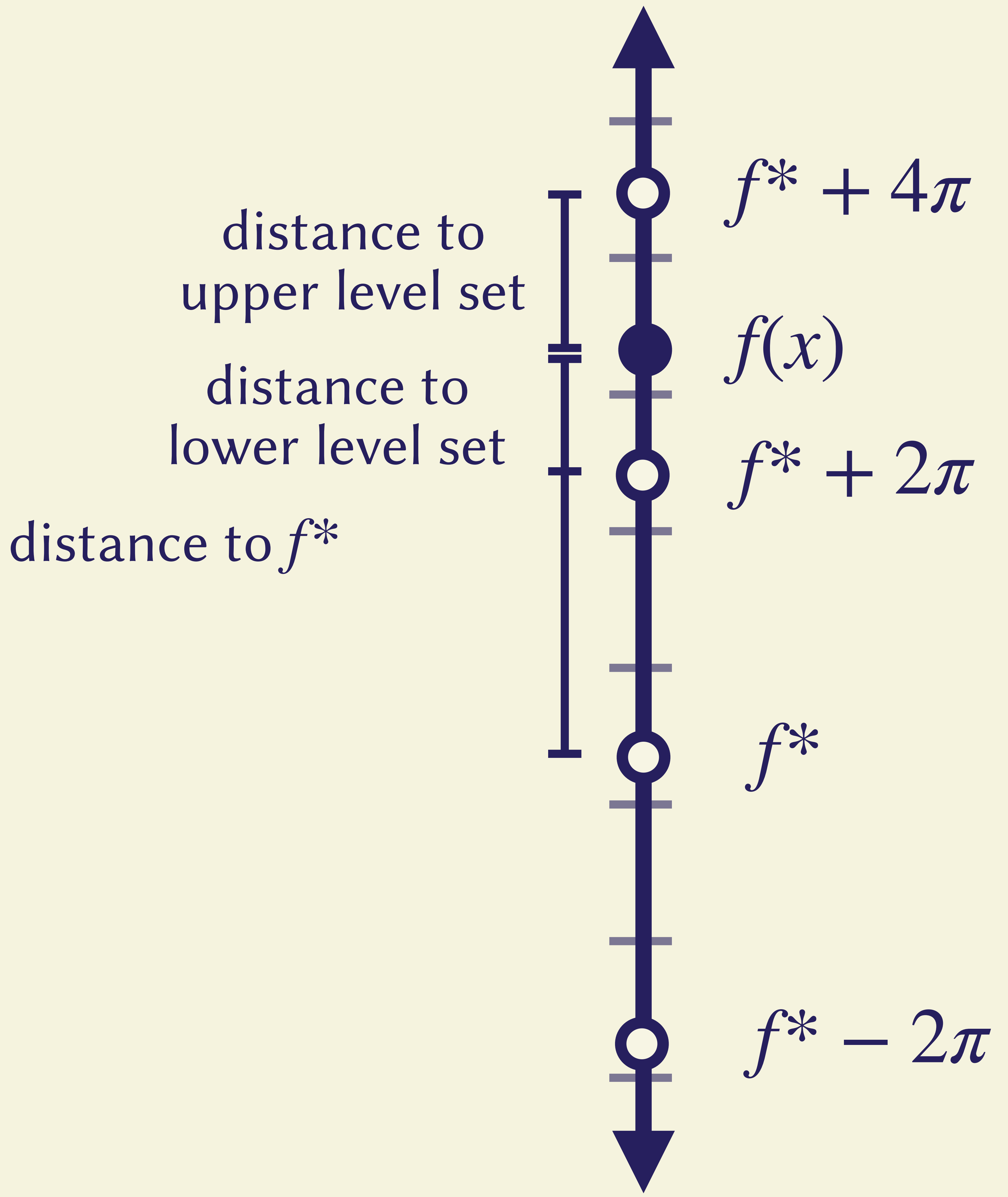
Angle-valued functions → continuous functions



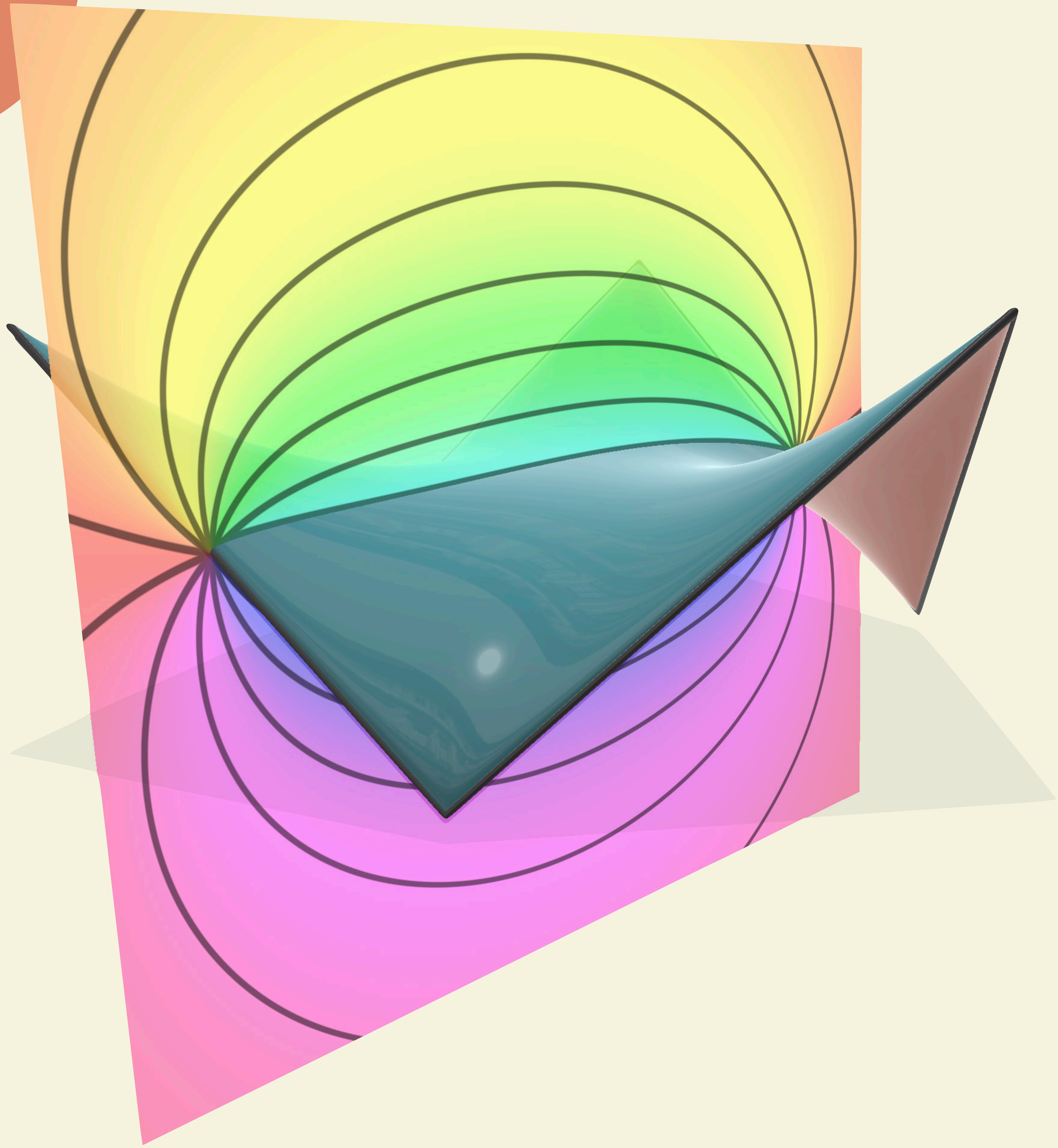
we never compute this lift explicitly—

Discontinuous Function
Continuous Lift

In practice: look for level sets above and below



Level sets of angle-valued functions can have boundaries



Knitting and Purling

Figures from [Signal *et al.* 2024]

"knit"

"purl"