

Implicit Minimal Surfaces for Bijective Correspondences

Discrete Geometric Structures

Monday, 3 August 2026

Mark Gillespie, UNIVERSITY OF UTAH

with



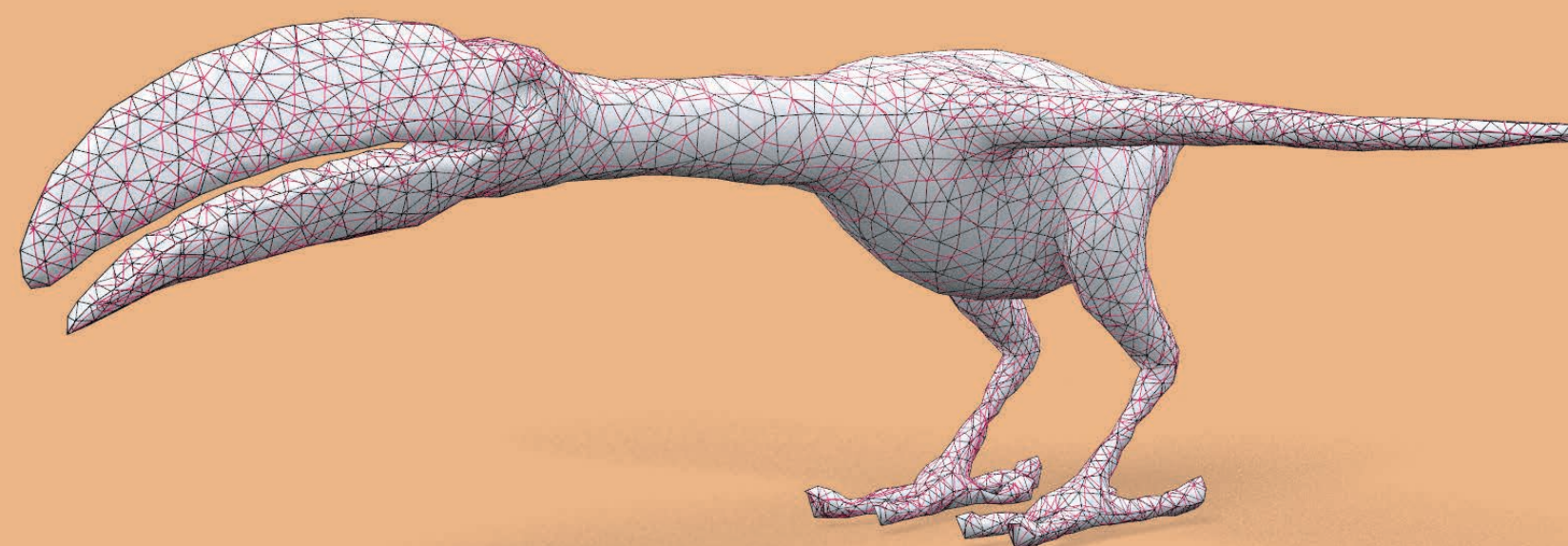
Etienne Corman, UNIVERSITÉ DE LORRAINE, CNRS, & LORIA.



Yousuf Soliman, SIDE EFFECTS SOFTWARE



Robin Magnet, INRIA & UNIVERSITÉ PARIS CITÉ

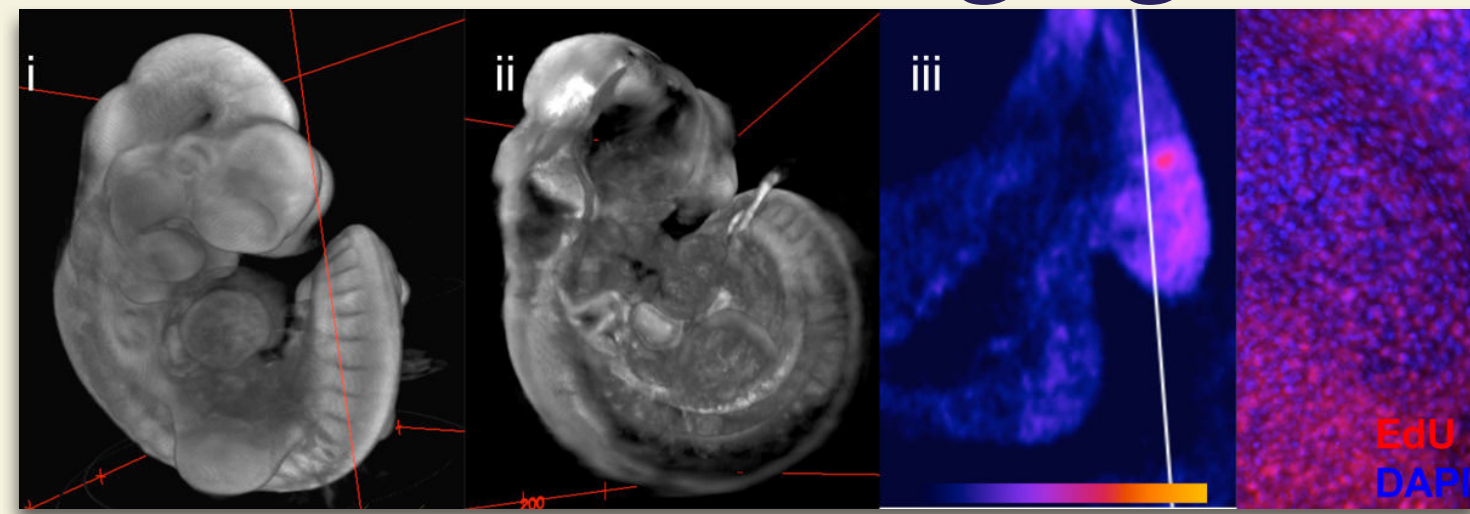


Problem: Computing Bijections Between Oriented Surfaces

formulate in terms of discrete complex line bundles on $A \times B$

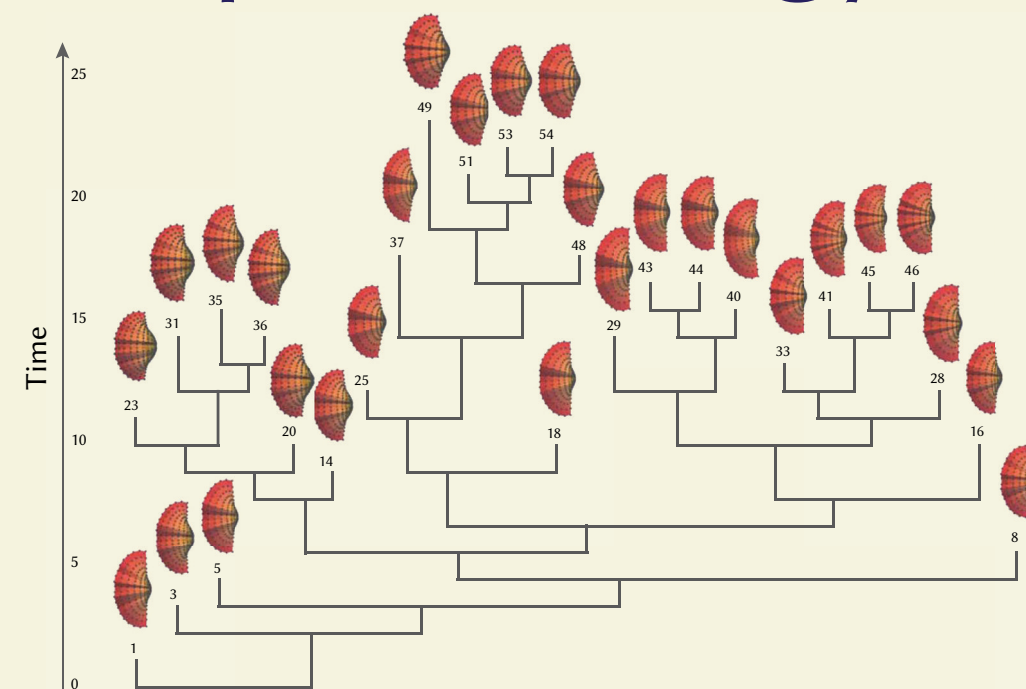
required to study data over collections of shapes

medical imaging

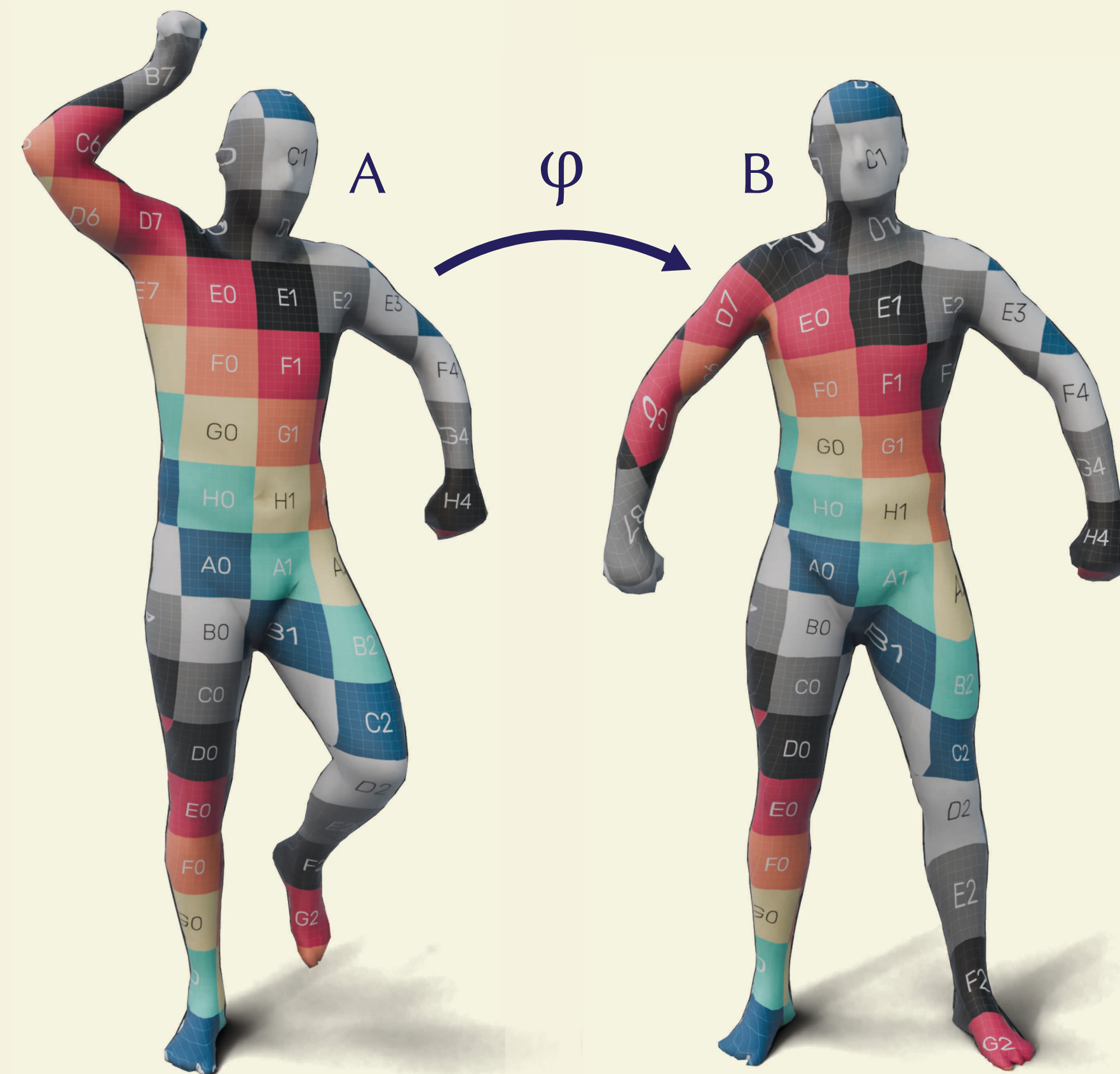


[Hallgrimsson et al. 2015]

paleontology



[Polly & Motz 2016]

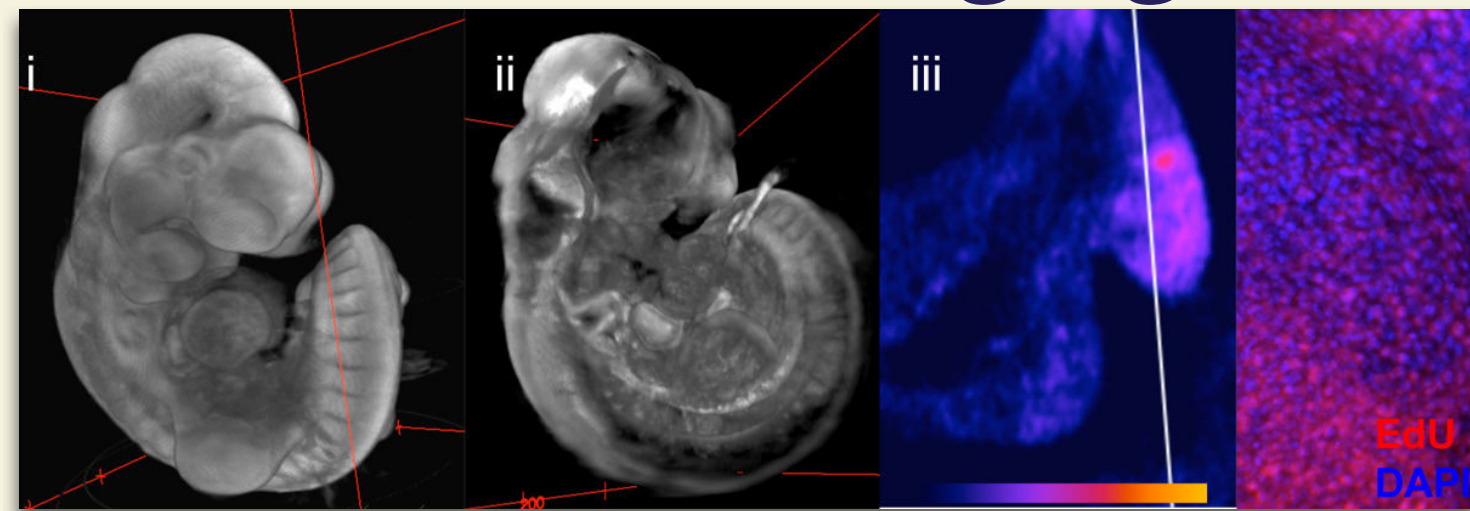


Problem: Computing Bijections Between Oriented Surfaces

formulate in terms of discrete complex line bundles on $A \times B$

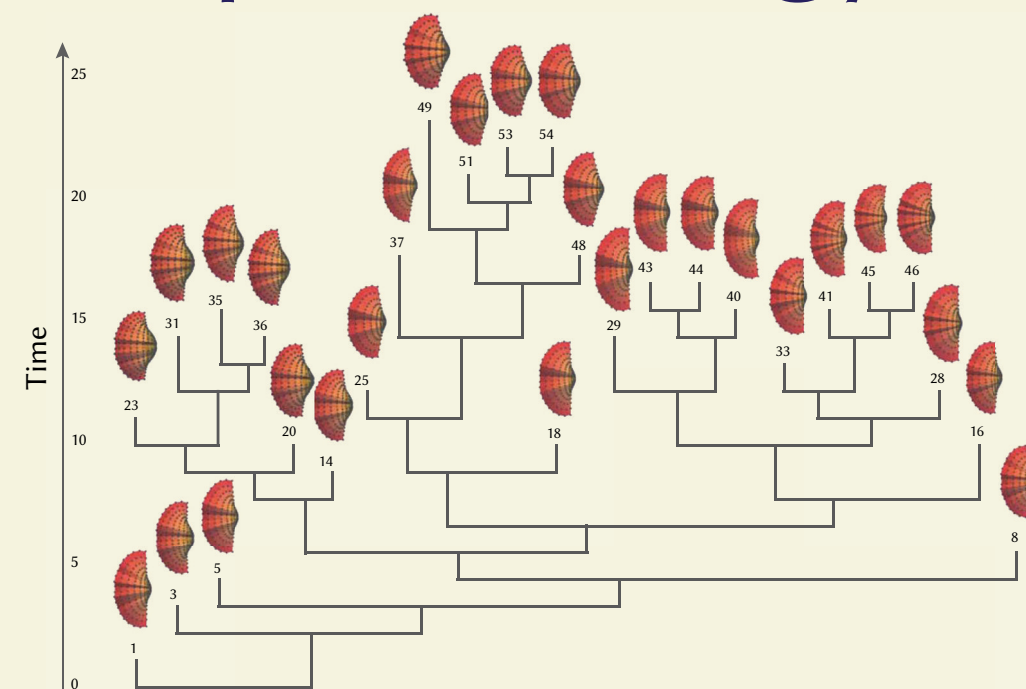
required to study data over collections of shapes

medical imaging

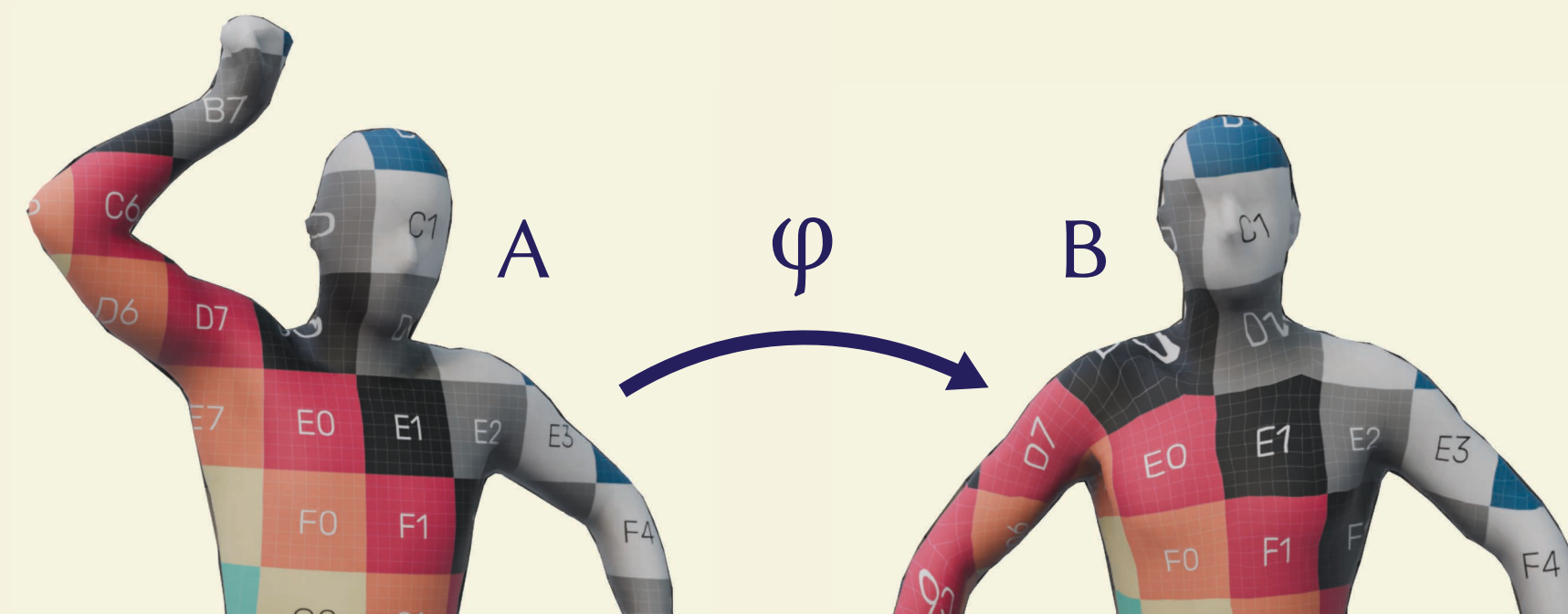


[Hallgrimsson et al. 2015]

paleontology

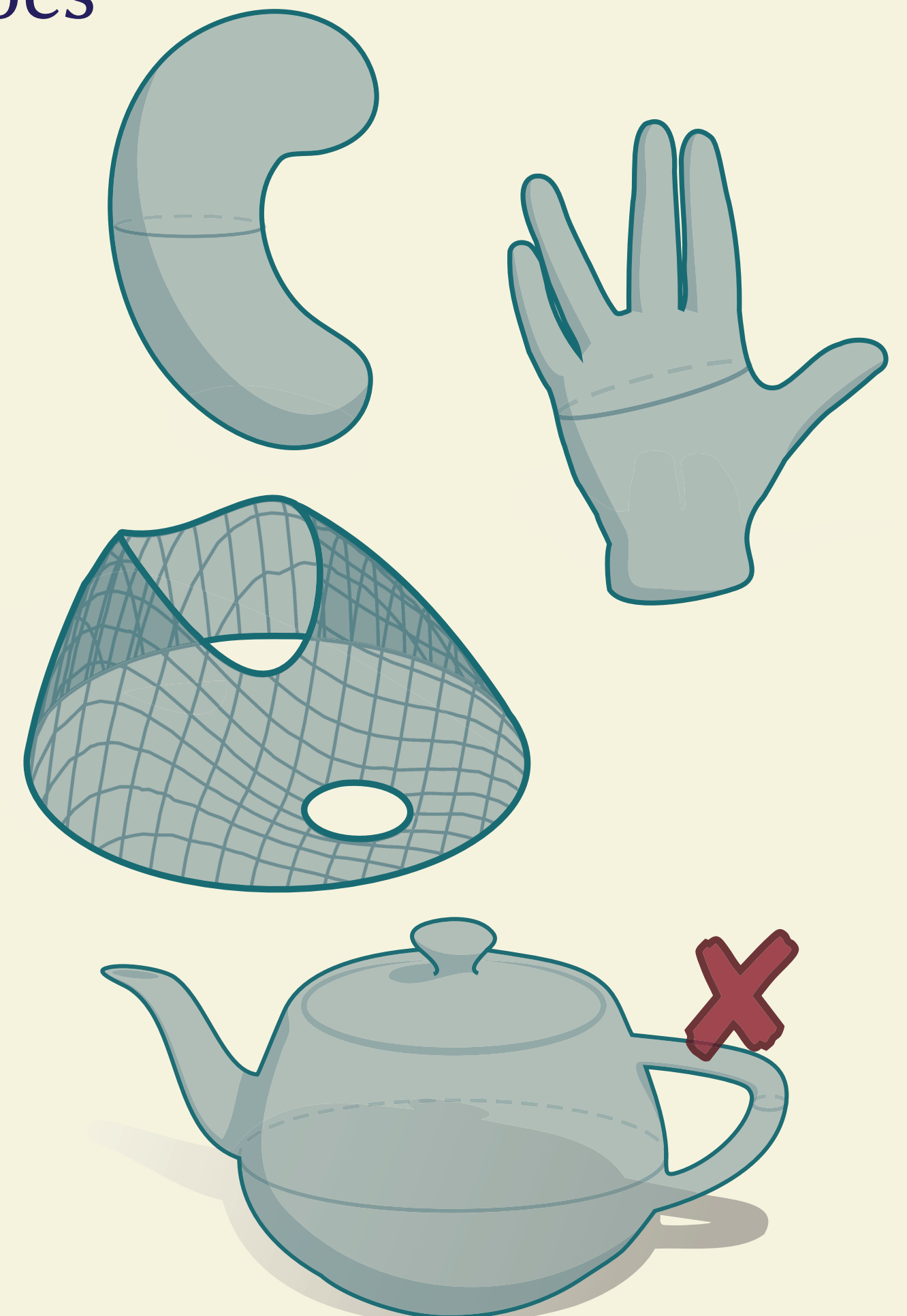


[Polly & Motz 2016]



SIMPLIFYING ASSUMPTION

Surfaces are topological spheres
(will discuss boundary near the end)
but no handles (yet)

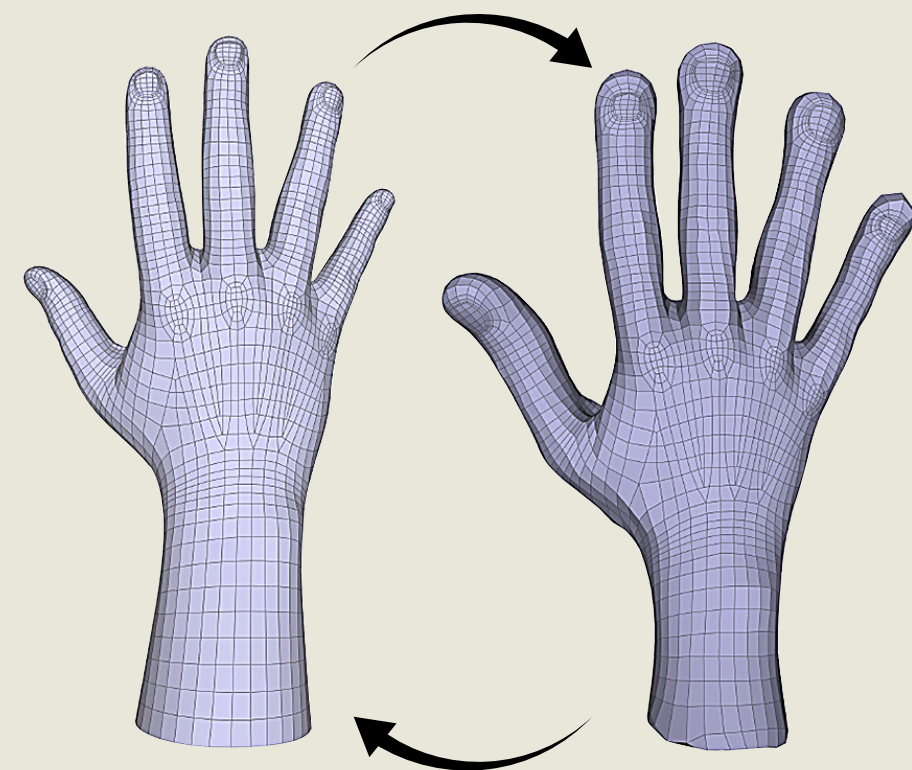


Existing Algorithms

SHAPE DEFORMATION

explicitly move mesh vertices

[Beg et al. 2005; Sharf et al. 2006; Huang et al. 2008; Li et al. 2008b; Tam et al. 2012; Eisenberger et al. 2019, 2020; Ezuz et al. 2019a]



Reversible Harmonic Maps (RHM)
[Ezuz et al. 2019b]

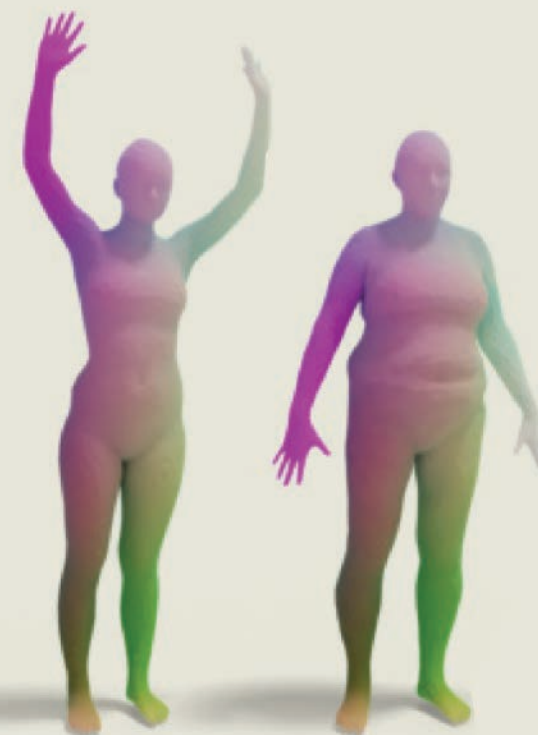
straightforward algorithms, *but*

- ✗ requires good initialization
- ✗ often requires user intervention

“FUZZY” RELAXATIONS

optimize $\varphi_ : L^2(A) \rightarrow L^2(B)$*

[Ovsjanikov et al. 2012, 2017; Ren et al. 2021; Litany et al. 2016; Solomon et al. 2012; Mandad et al. 2017; Brifault et al. 2025]



ZoomOut
[Melzi et al. 2019]

simpler to optimize, *but*:

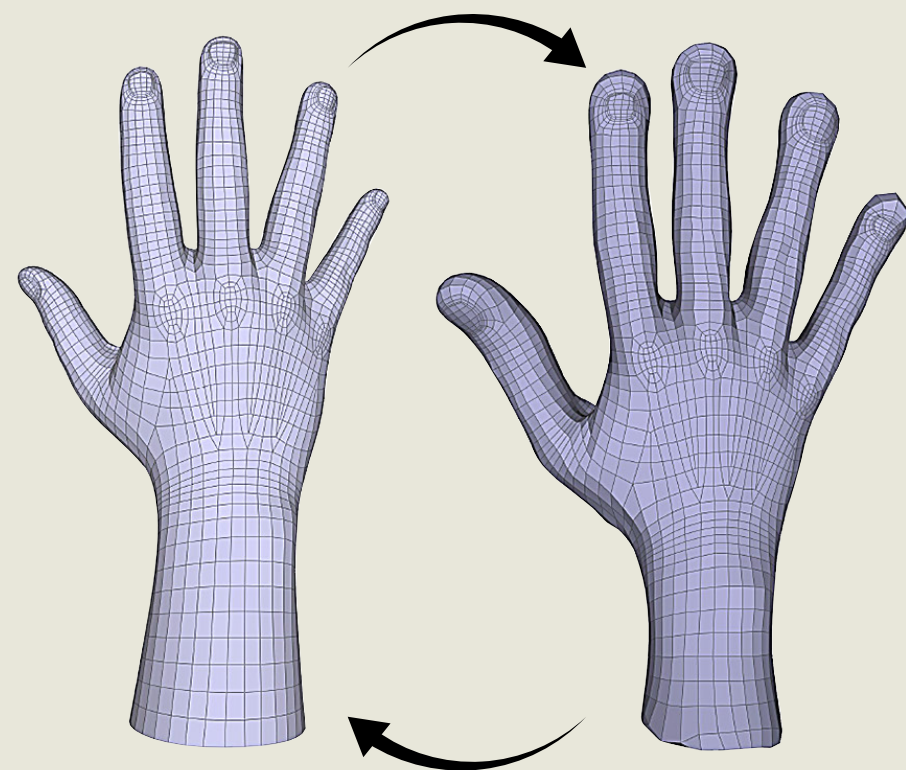
- ✗ can collapse features
- ✗ may struggle with symmetries

Existing Algorithms

SHAPE DEFORMATION

explicitly move mesh vertices

[Beg et al. 2005; Sharf et al. 2006; Huang et al. 2008; Li et al. 2008b; Tam et al. 2012; Eisenberger et al. 2019, 2020; Ezuz et al. 2019a]



Reversible Harmonic Maps (RHM)
[Ezuz et al. 2019b]

straightforward algorithms, *but*

- ✗ requires good initialization
- ✗ often requires user intervention

“FUZZY” RELAXATIONS

optimize $\varphi_ : L^2(A) \rightarrow L^2(B)$*

[Ovsjanikov et al. 2012, 2017; Ren et al. 2021; Litany et al. 2016; Solomon et al. 2012; Mandad et al. 2017; Brifault et al. 2025]



ZoomOut
[Melzi et al. 2019]

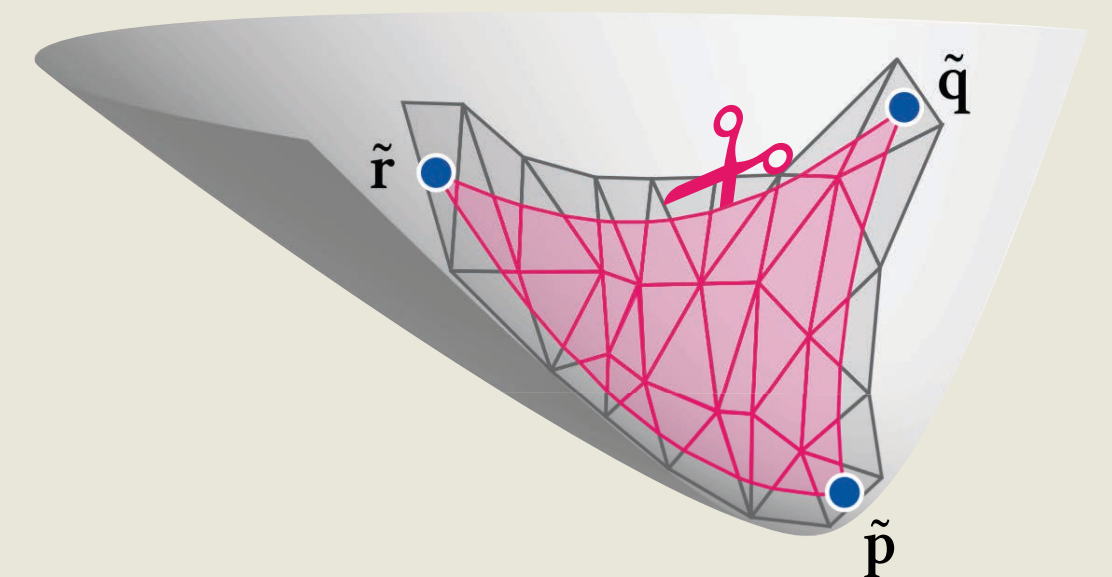
simpler to optimize, *but*:

- ✗ can collapse features
- ✗ may struggle with symmetries

EXPLICIT MESH MAPS

optimize explicit common subdivision

[Kanai et al. 1997; Litke et al. 2005; Aigerman et al. 2014–2017; Schreiner et al. 2004; Kim et al. 2011; Michikawa et al. 2001; Peng & Timalaena 2016; Kraevoy & Sheffer 2004; Takayama 2022]



Inter-Surface Maps via Constant-Curvature Metrics (ISM)
[Schmidt et al. 2020]

bijective by construction, *but*:

- ✗ requires bijective initialization
- ✗ requires complicated mesh mutation

Existing Algorithms

SHAPE DEFORMATION

explicitly deform mesh

[Beg et al. 2005; Sharf et al. 2008; Li et al. 2008b; Tam et al.

“FUZZY” RELAXATIONS

minimize $\int_2(A) + \int_2(B)$

EXPLICIT MESH MAPS

define explicit common subdivision

[1997; Litke et al. 2005; Aigerman et al. 2007; Schreiner et al. 2004; Kim et al. 2022]

Our implicit complex line bundle formulation combines the strengths of these methods:

- straightforward algorithm: built on familiar operators from tangent vector field processing
- energy is simple to optimize: can "untangle" low-quality, non-bijective initializations
- orientation-preserving by construction
- minimizer provides map between meshes without mesh mutation

straightforward algorithms, *but*

- ✗ requires good initialization
- ✗ often requires user intervention

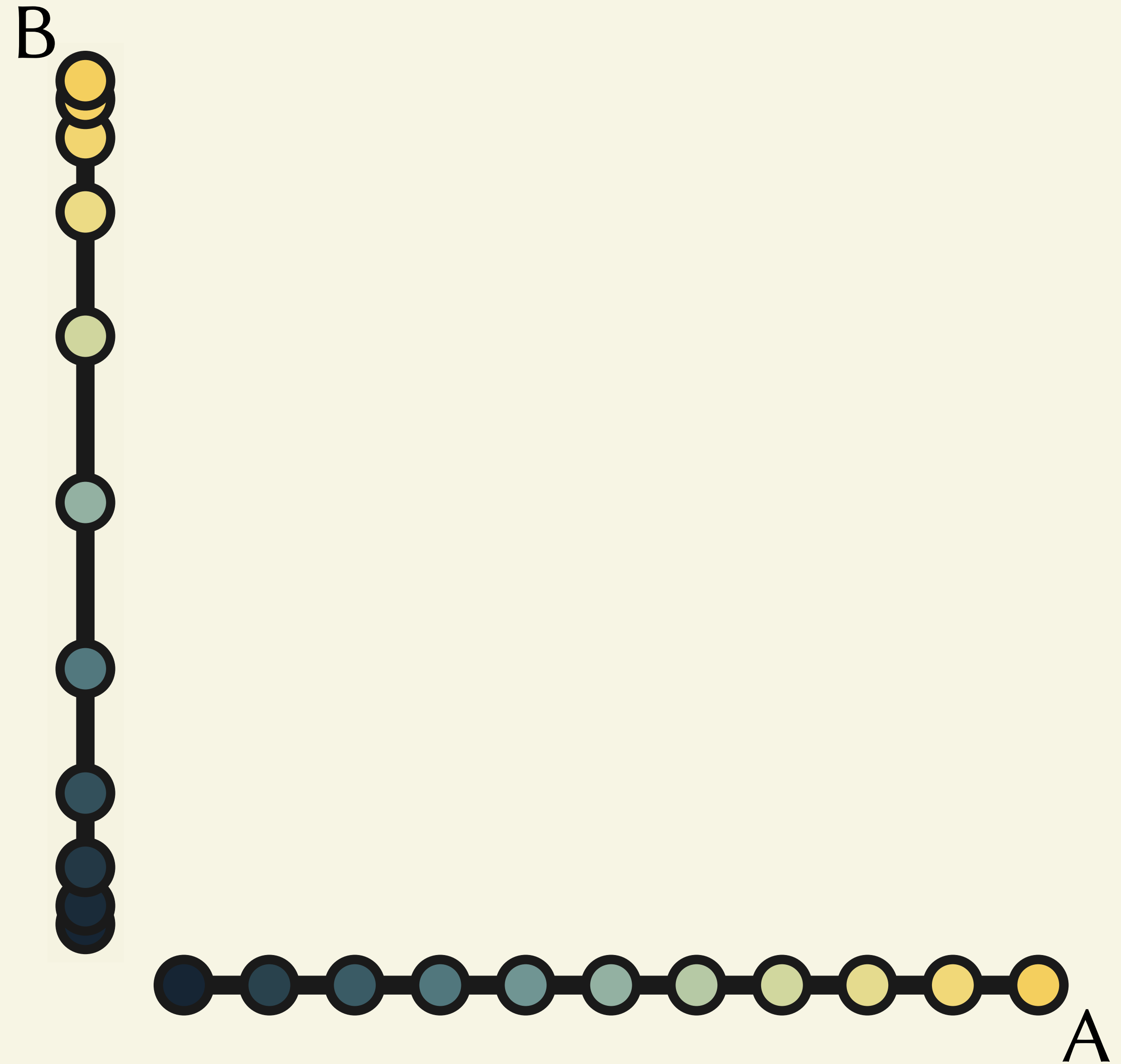
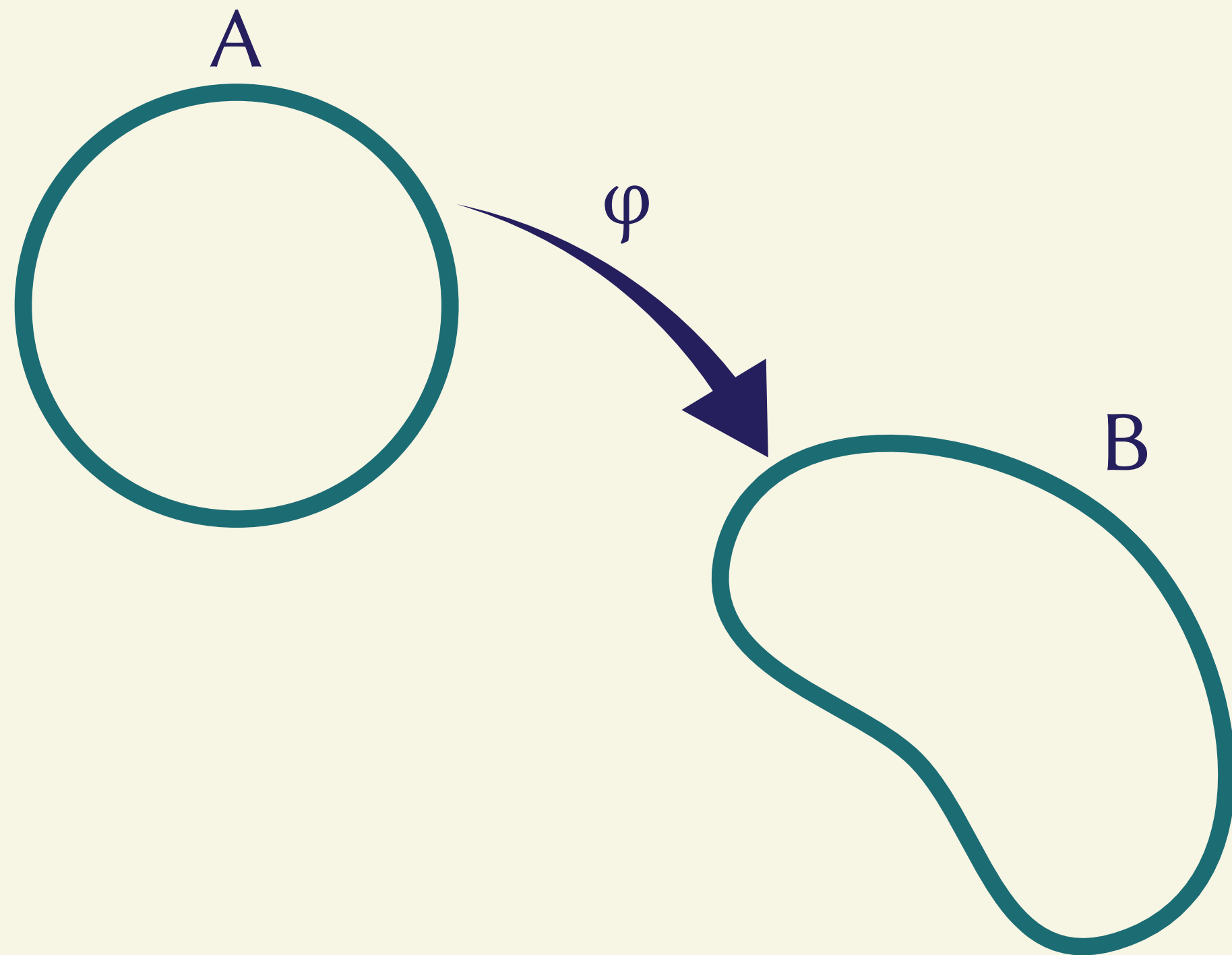
simpler to optimize, *but:*

- ✗ can collapse features
- ✗ may struggle with symmetries

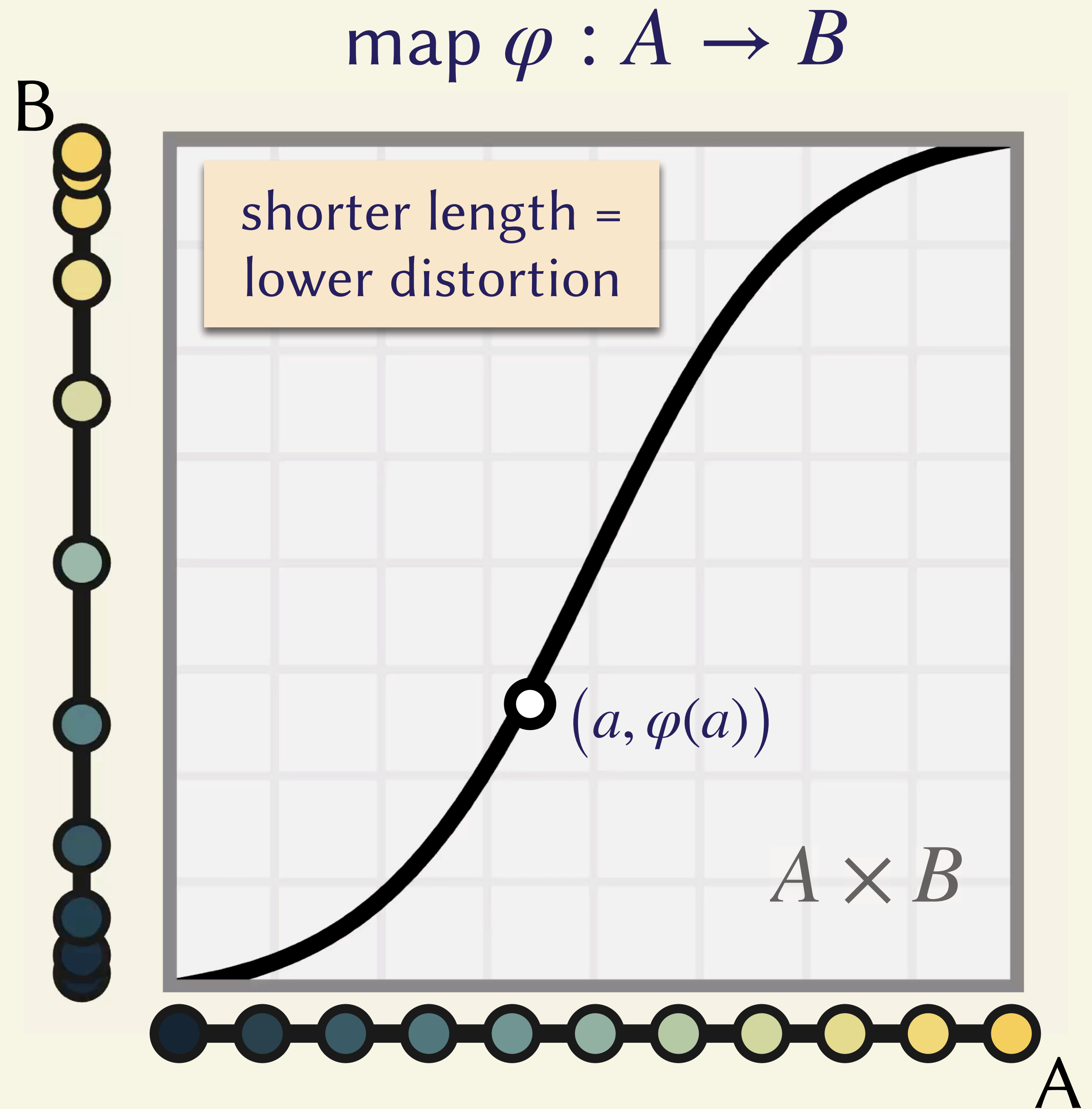
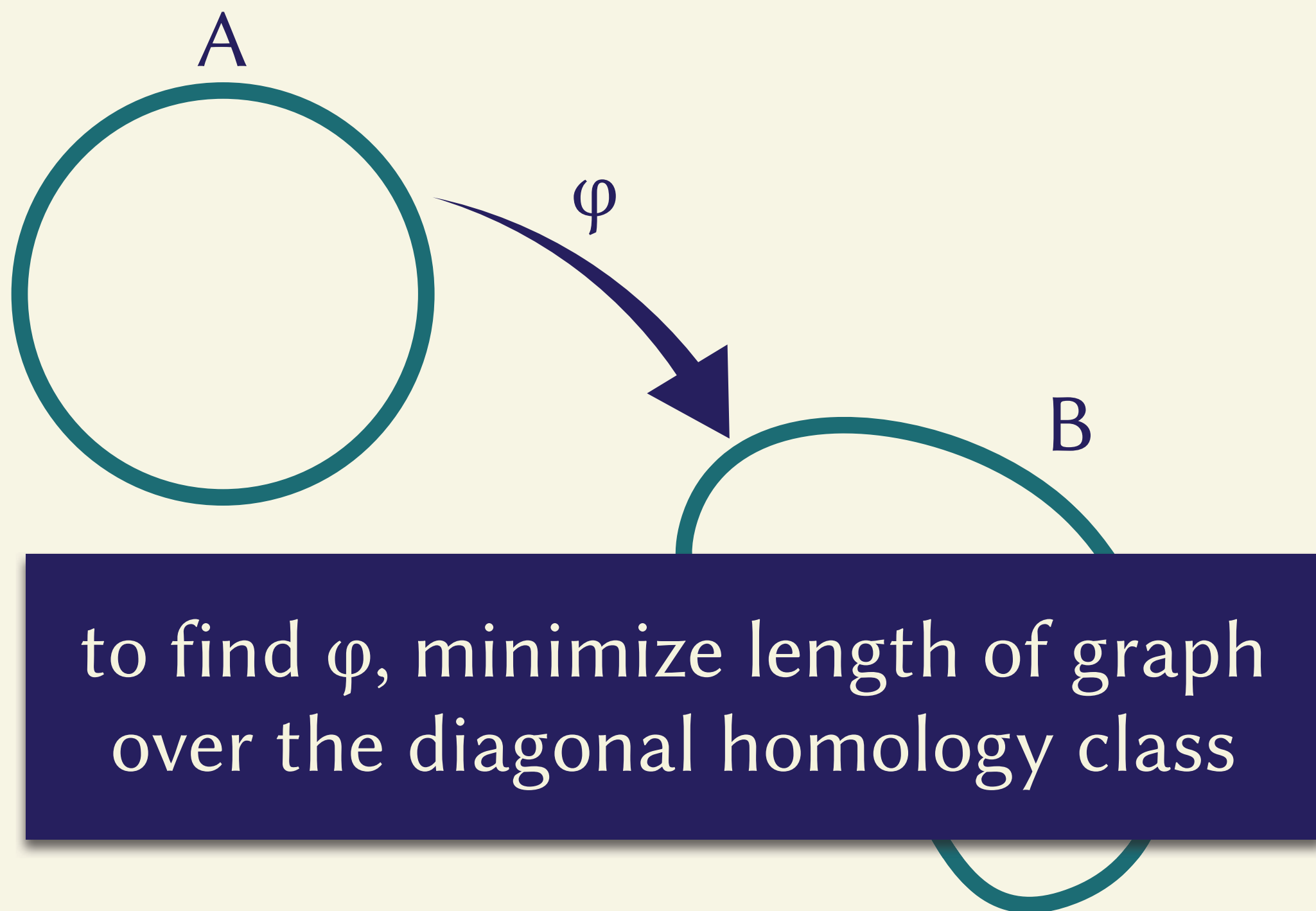
bijective by construction, *but:*

- ✗ requires bijective initialization
- ✗ requires complicated mesh mutation

Example: 1D Correspondence



Example: 1D Correspondence



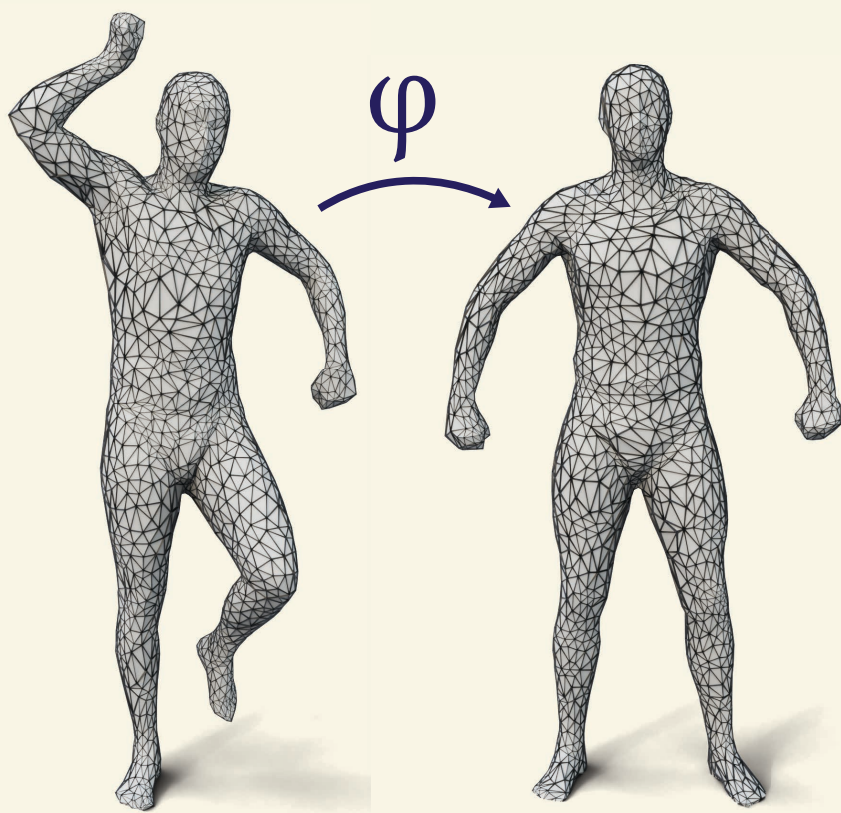
Mapping between 2D surfaces

to find φ , minimize area
of Σ over the diagonal
homology class
 $[\Delta] = [\text{pt}_A \times B + A \times \text{pt}_B]$

homology class of
orientation-preserving maps!

What does $\text{Area}[\Sigma_\varphi]$
measure?

$$\varphi : A \rightarrow B$$



$$\Sigma_\varphi = \{(a, \varphi(a))\} \subset A \times B$$

$$\text{Area}[\Sigma_\varphi] = \text{Area}[(\text{Id} \times \varphi)(A)]$$

$$= \int_A \sqrt{\det(\mathbb{I} + g_A^{-1} \varphi^* g_B)} \, dA$$

$$= \int_A \sqrt{(1 + \sigma_1^2)(1 + \sigma_2^2)} \, dA$$

(σ_1, σ_2 singular values of $d\varphi$)

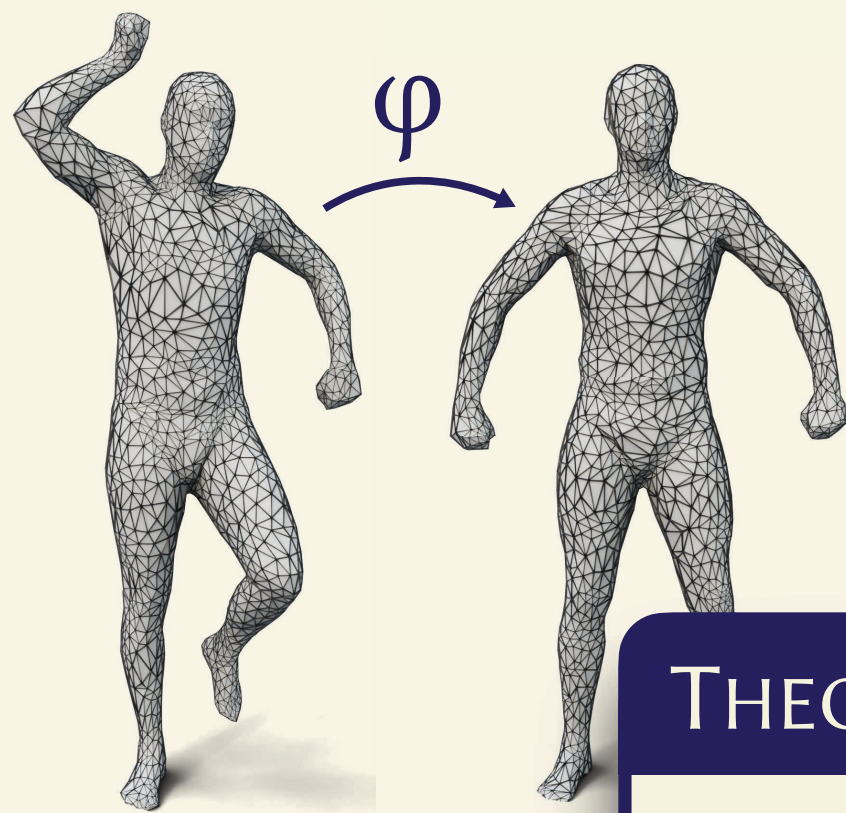
Mapping between 2D surfaces

to find φ , minimize area
of Σ over the diagonal
homology class
 $[\Delta] = [\text{pt}_A \times B + A \times \text{pt}_B]$

homology class of
orientation-preserving maps!

$$\varphi : A \rightarrow B$$

$$\Sigma_\varphi = \{(a, \varphi(a))\} \subset A \times B$$



$$\text{Area}[\Sigma_\varphi] = \int_A \sqrt{(1 + \sigma_1^2)(1 + \sigma_2^2)} \, dA$$

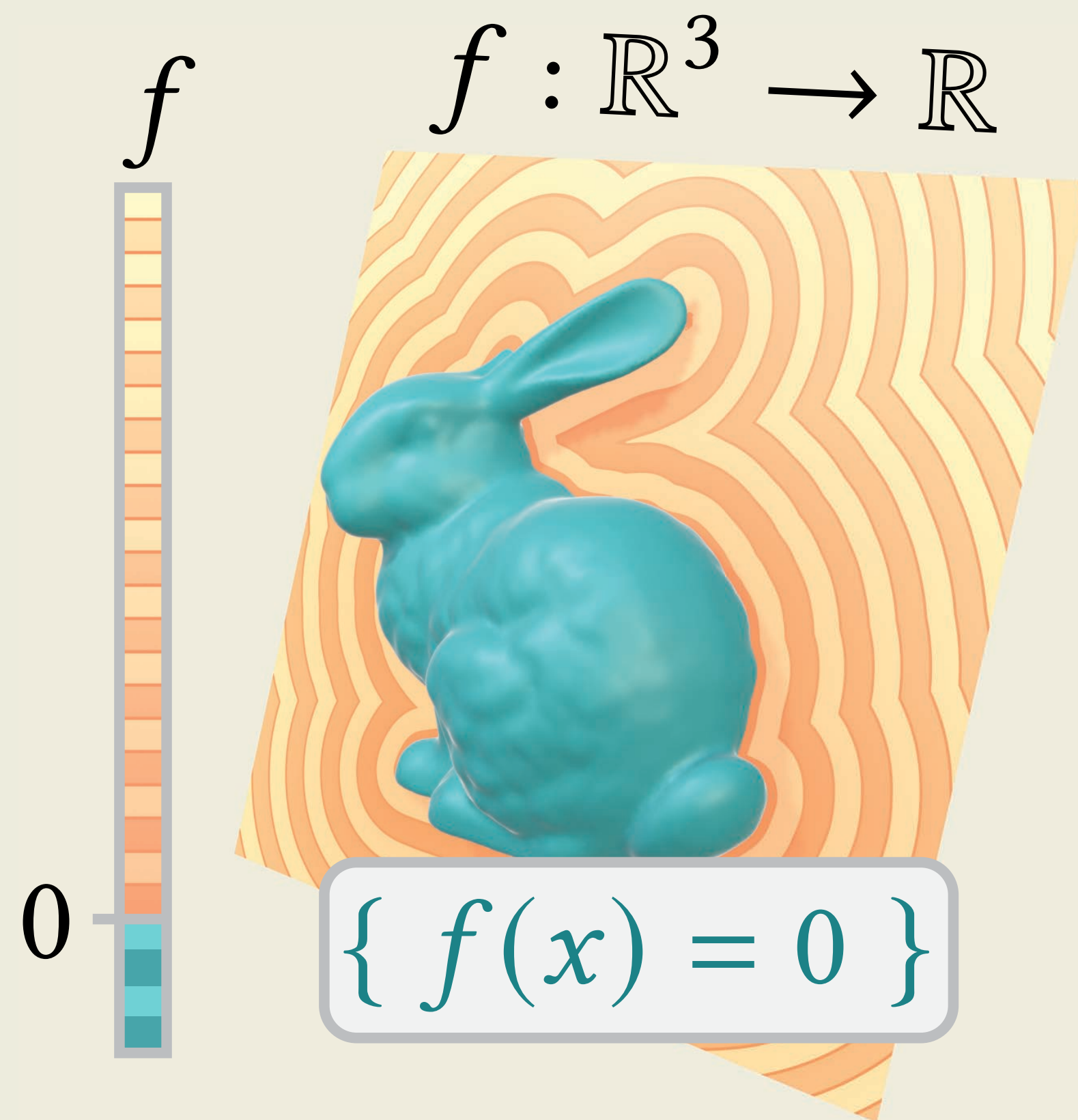
$(\sigma_1, \sigma_2 \text{ singular values of } d\varphi)$

THEOREM [CSMG 2026]:

$$\text{Area}[A] + \int_A \overset{\text{AREA DISTORTION}}{|\det d\varphi|} \, dA \leq \text{Area}[\Sigma_\varphi] \leq \text{Area}[A] + \int_A \overset{\text{DIRICHLET ENERGY}}{\frac{1}{2} |d\varphi|^2} \, dA$$

Implicit Encoding

Easy way of encoding a 2D surface in 3D space

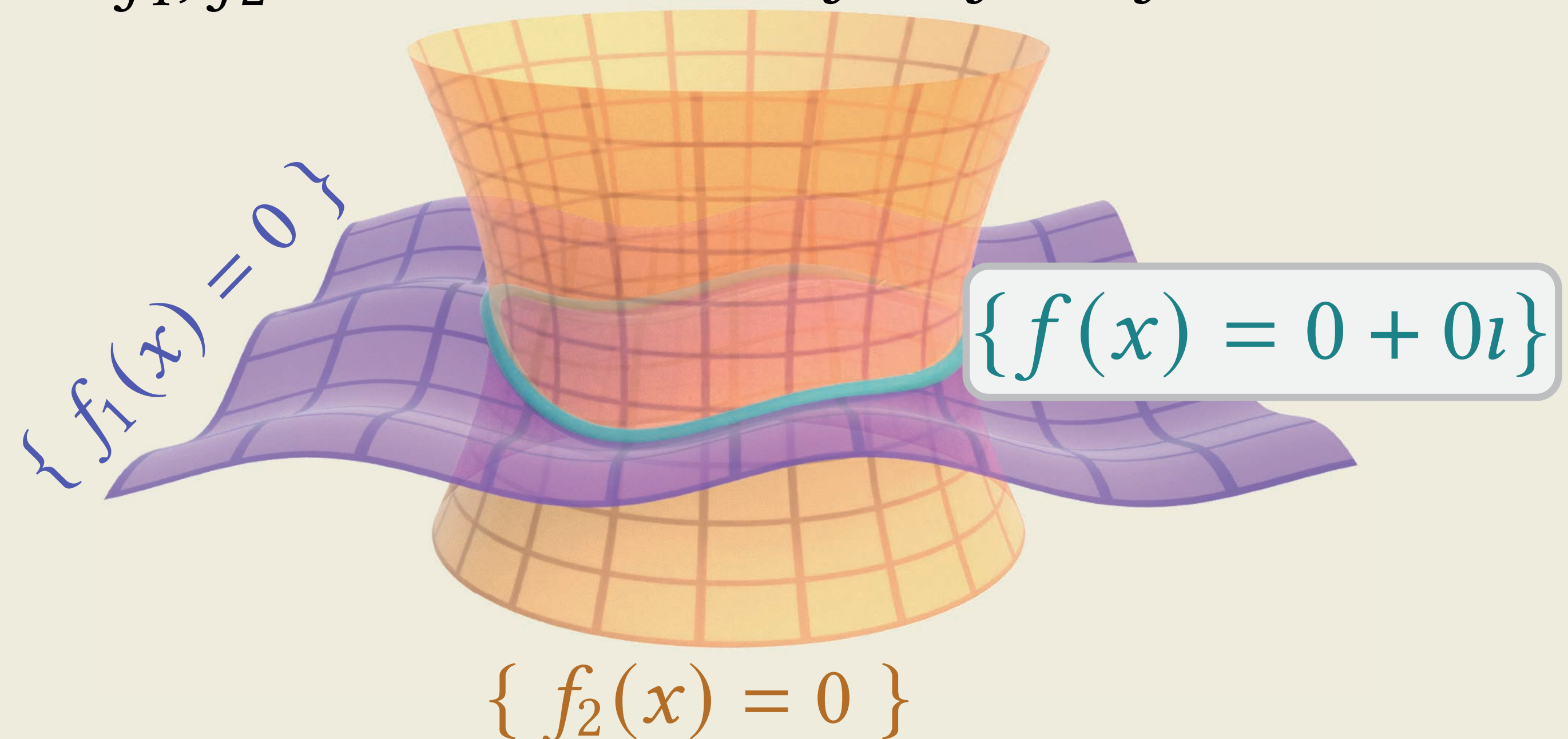


How about a 2D surface in a 4D space?

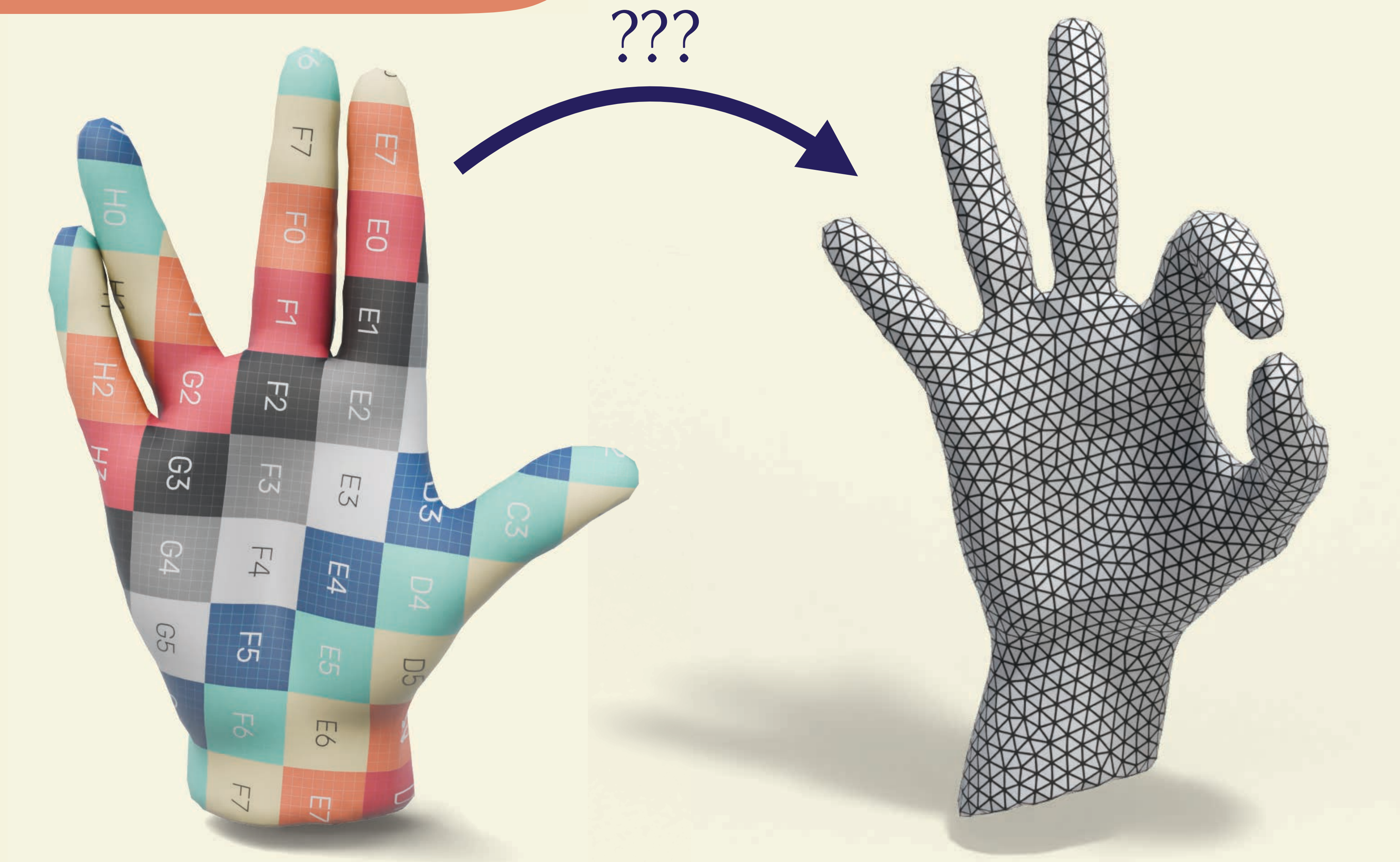
$\implies$ encode as the zero level set
of a *complex* function

(more correctly: a section of a
complex line bundle)

$$f_1, f_2 : \mathbb{R}^3 \rightarrow \mathbb{R} \iff f = f_1 + \imath f_2 : \mathbb{R}^3 \rightarrow \mathbb{C}$$



Algorithm Sketch



Algorithm Sketch

I. CONSTRUCT
COMPLEX LINE BUNDLES



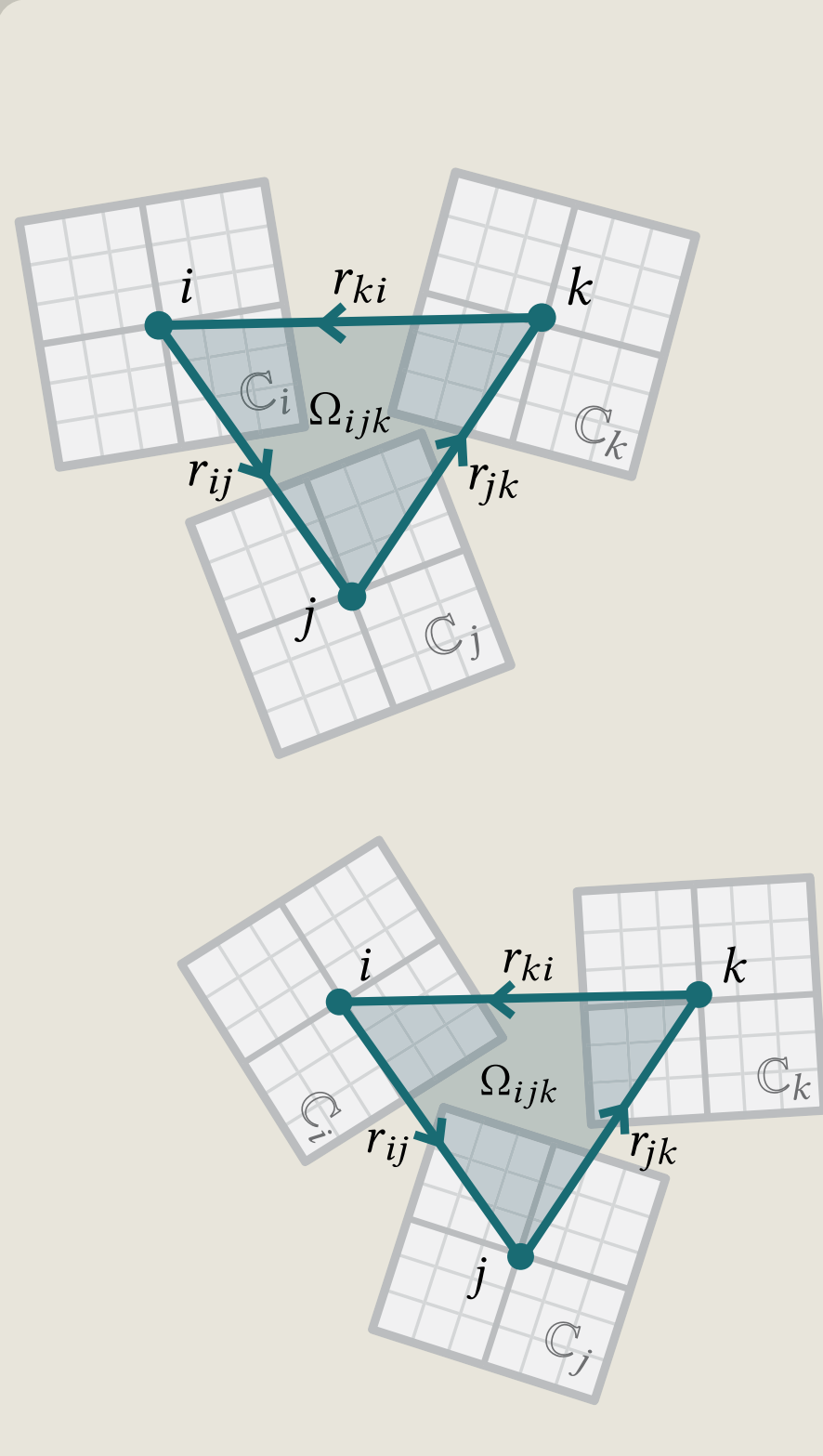
II. FIND AN
INITIAL MAP



III. FIT IMPLICIT
MAP

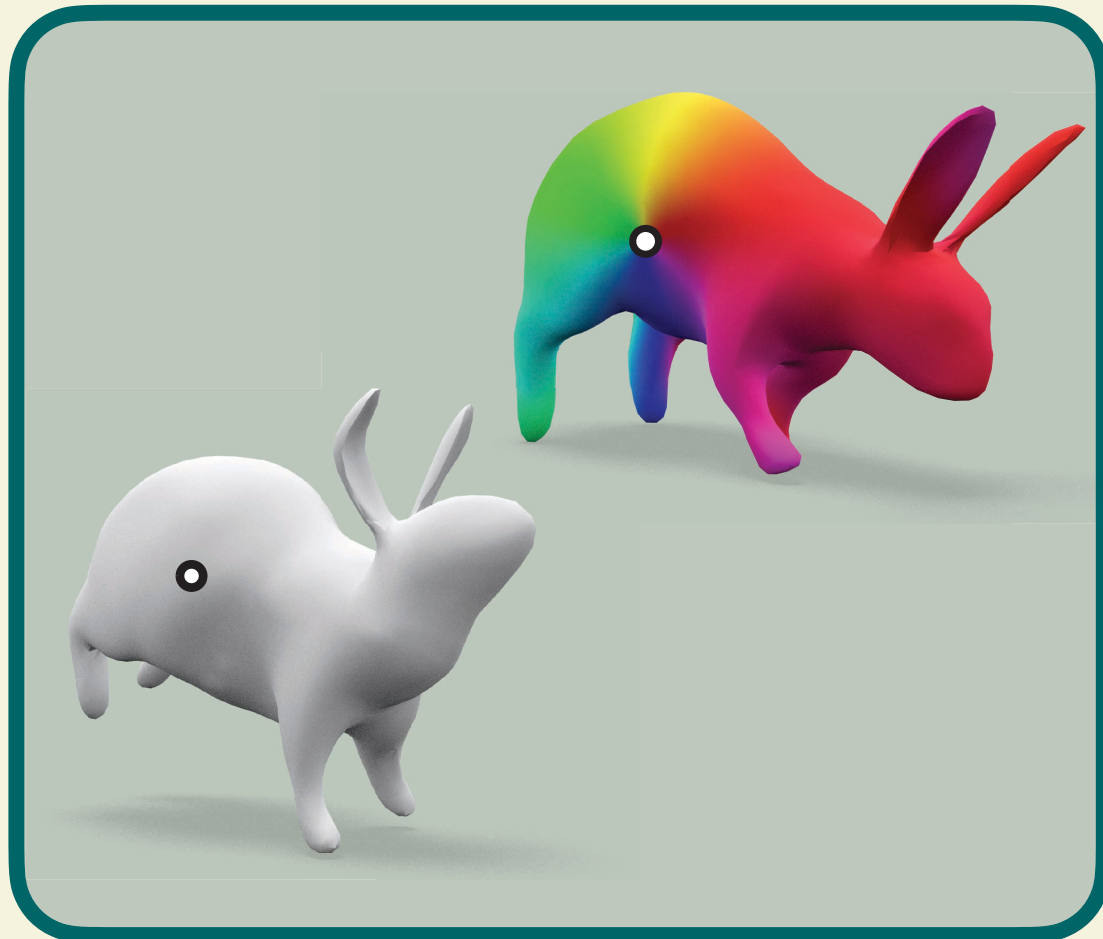


IV. OPTIMIZE
IMPLICIT MAP



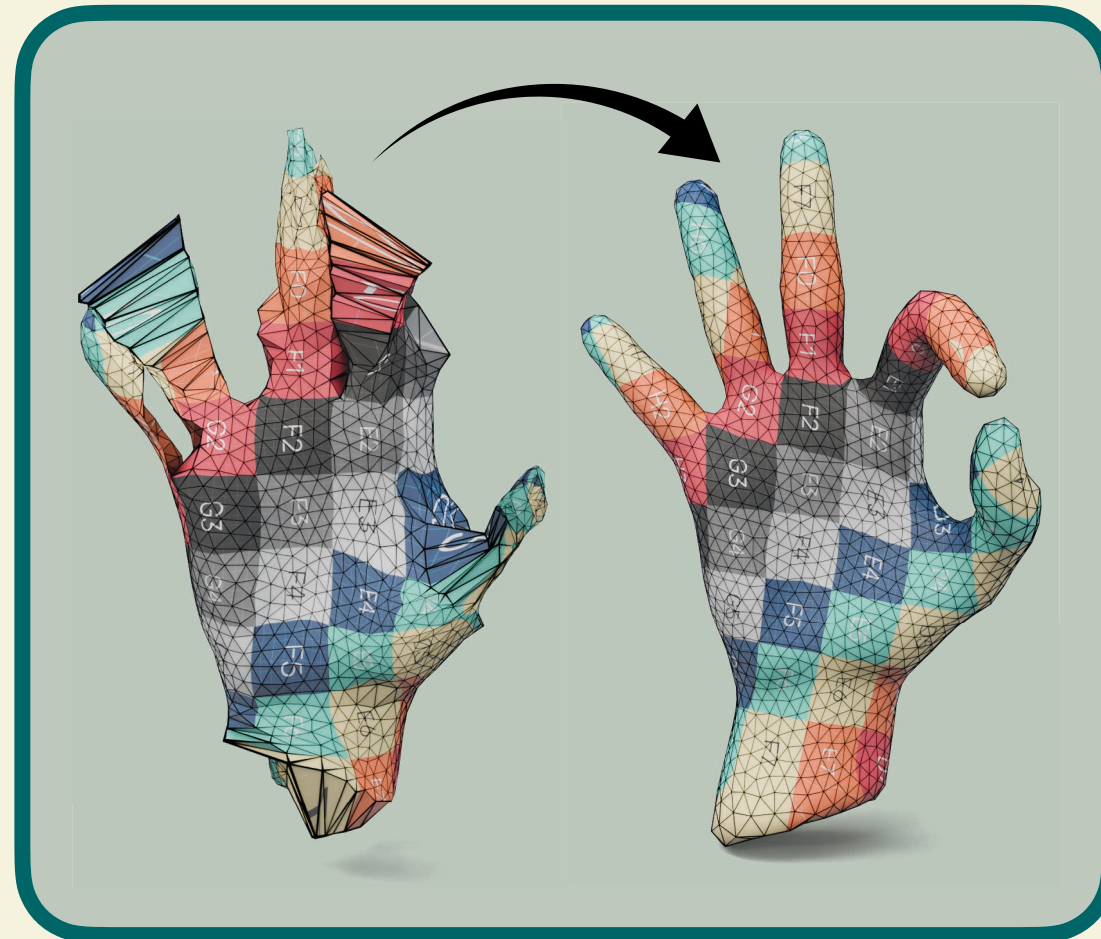
Talk Outline

I. IMPLICIT ENCODING



represent map as a section
of a complex line bundle

II. MAP OPTIMIZATION



complex line bundle
formulation leads to
simple optimization
problem

III. EXAMPLES



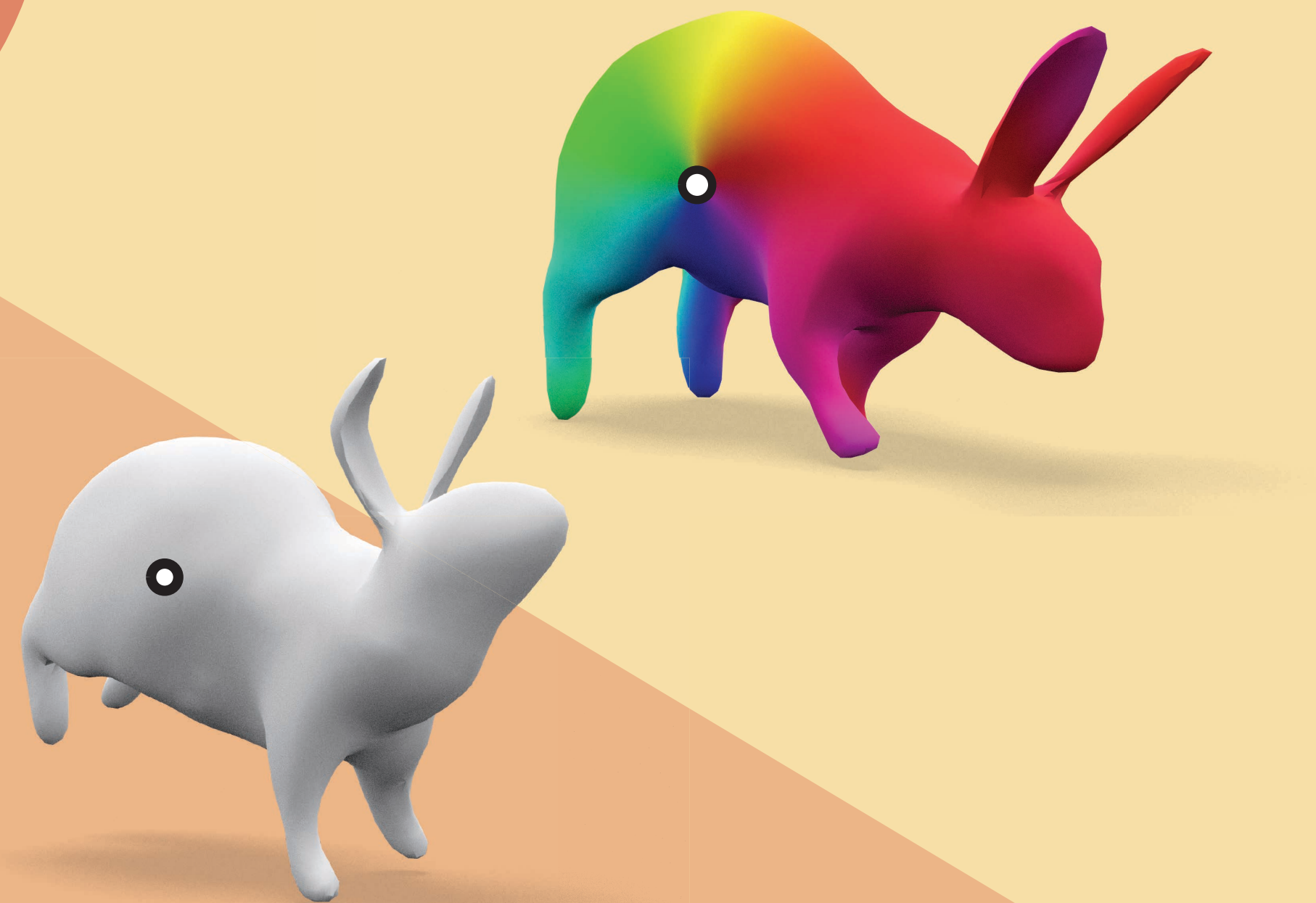
robustness & quality
advantages

IV. OPEN QUESTIONS



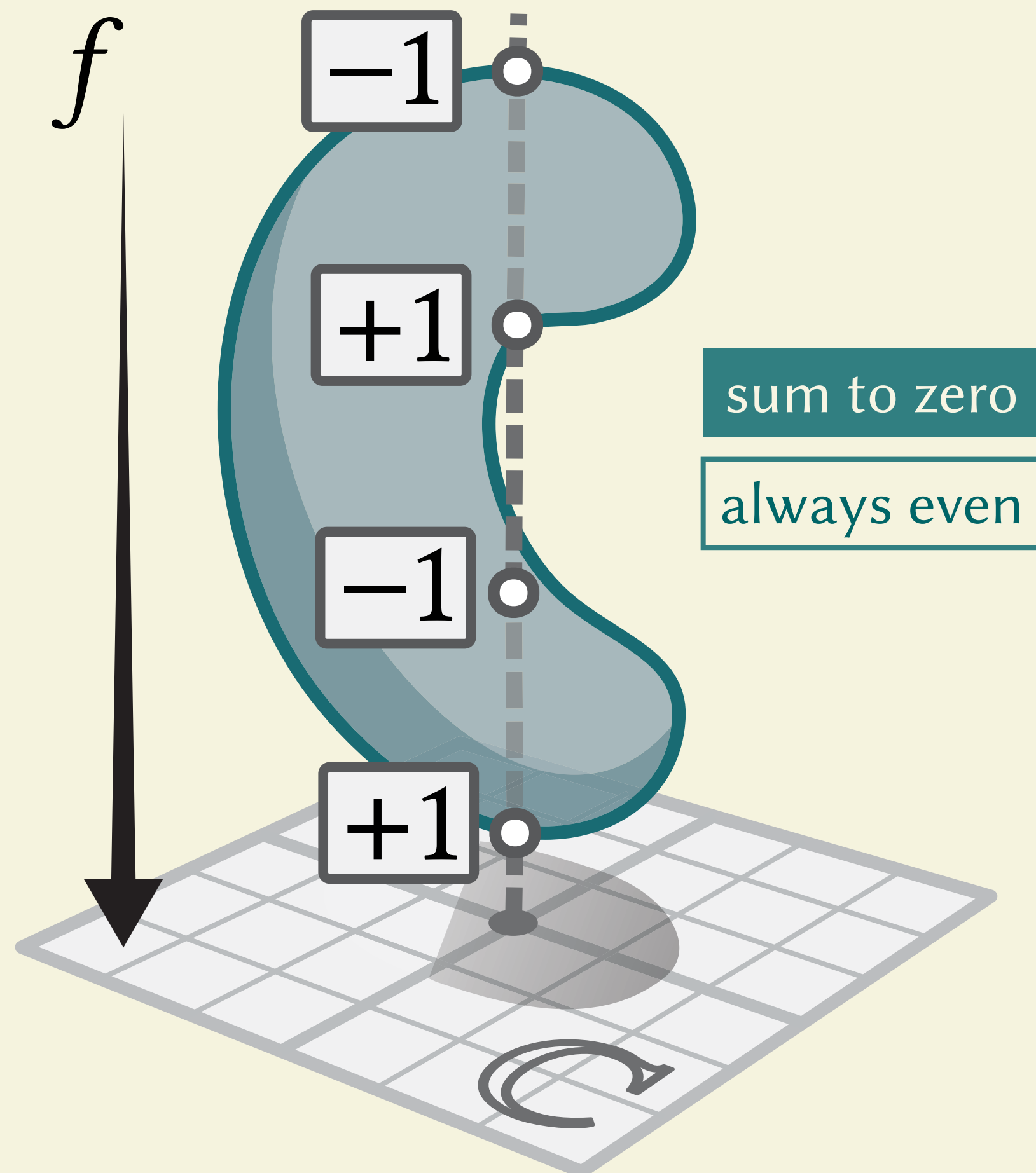
1

Implicit Encoding of Surfaces

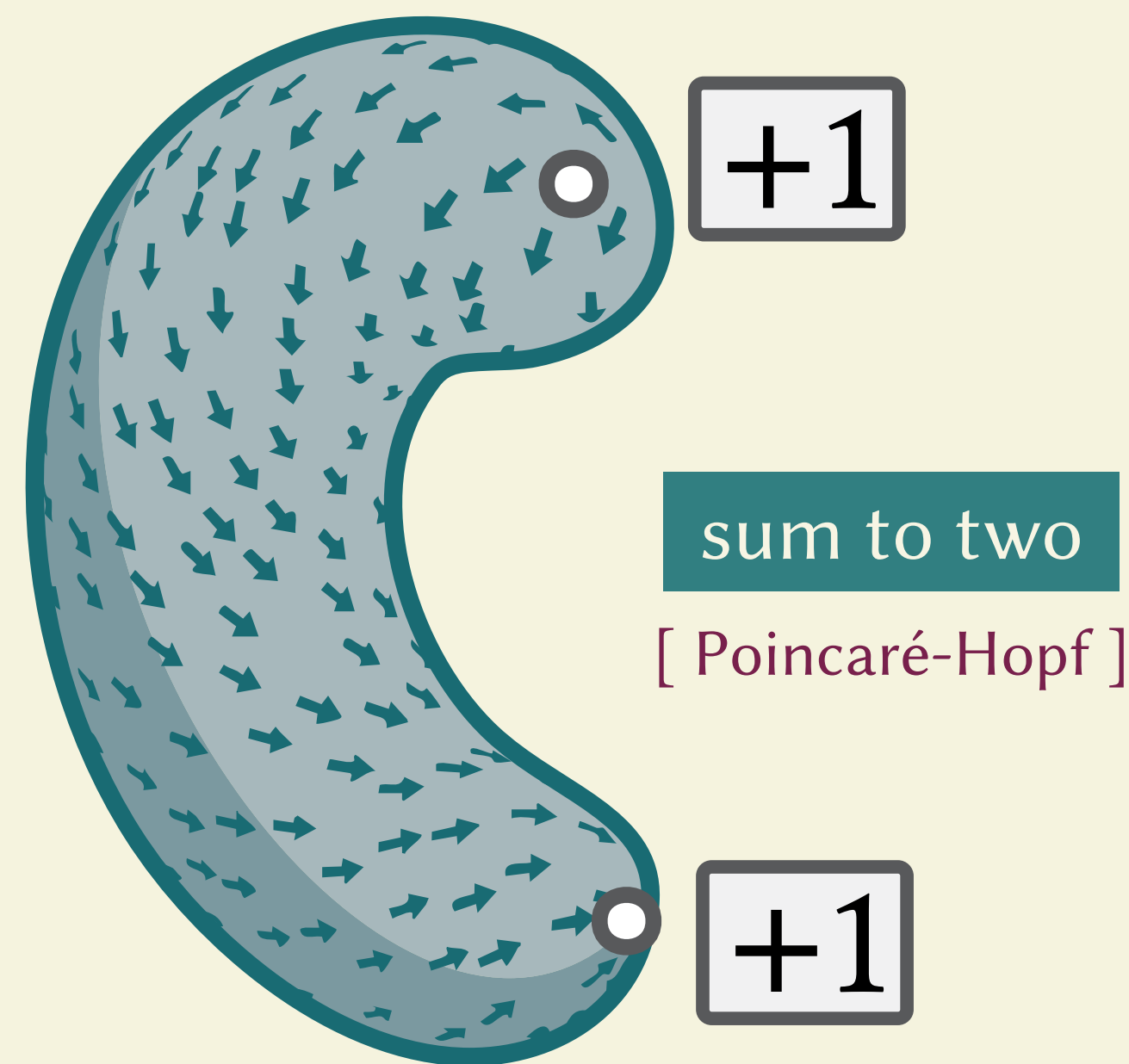


Prelude: Implicit Encoding of Points

Option 1: complex functions



Option 2: vector fields



Choice of representation
constrains which point
sets we can represent

space of
complex functions

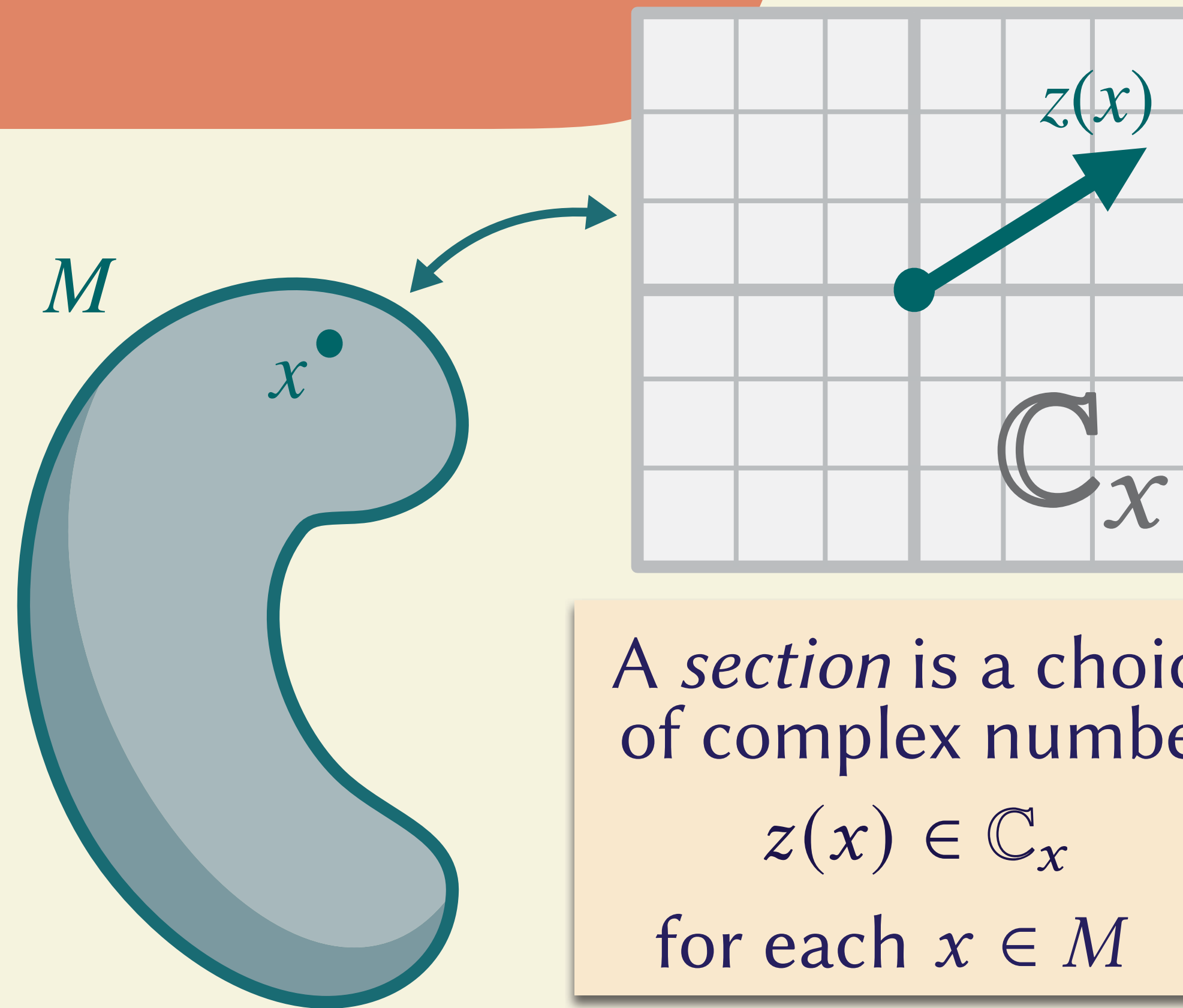
space of
vector fields

examples of
complex line bundles

global structure of
complex line bundle

$\iff$
constraints on
point set

Complex line bundles



A *section* is a choice
of complex number

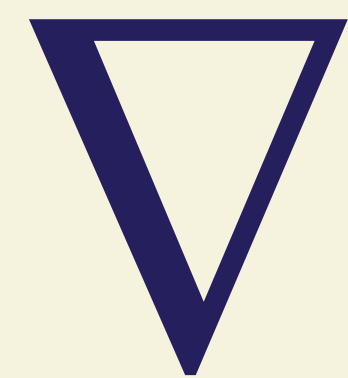
$$z(x) \in \mathbb{C}_x$$

for each $x \in M$

generalized
complex function /
vector field

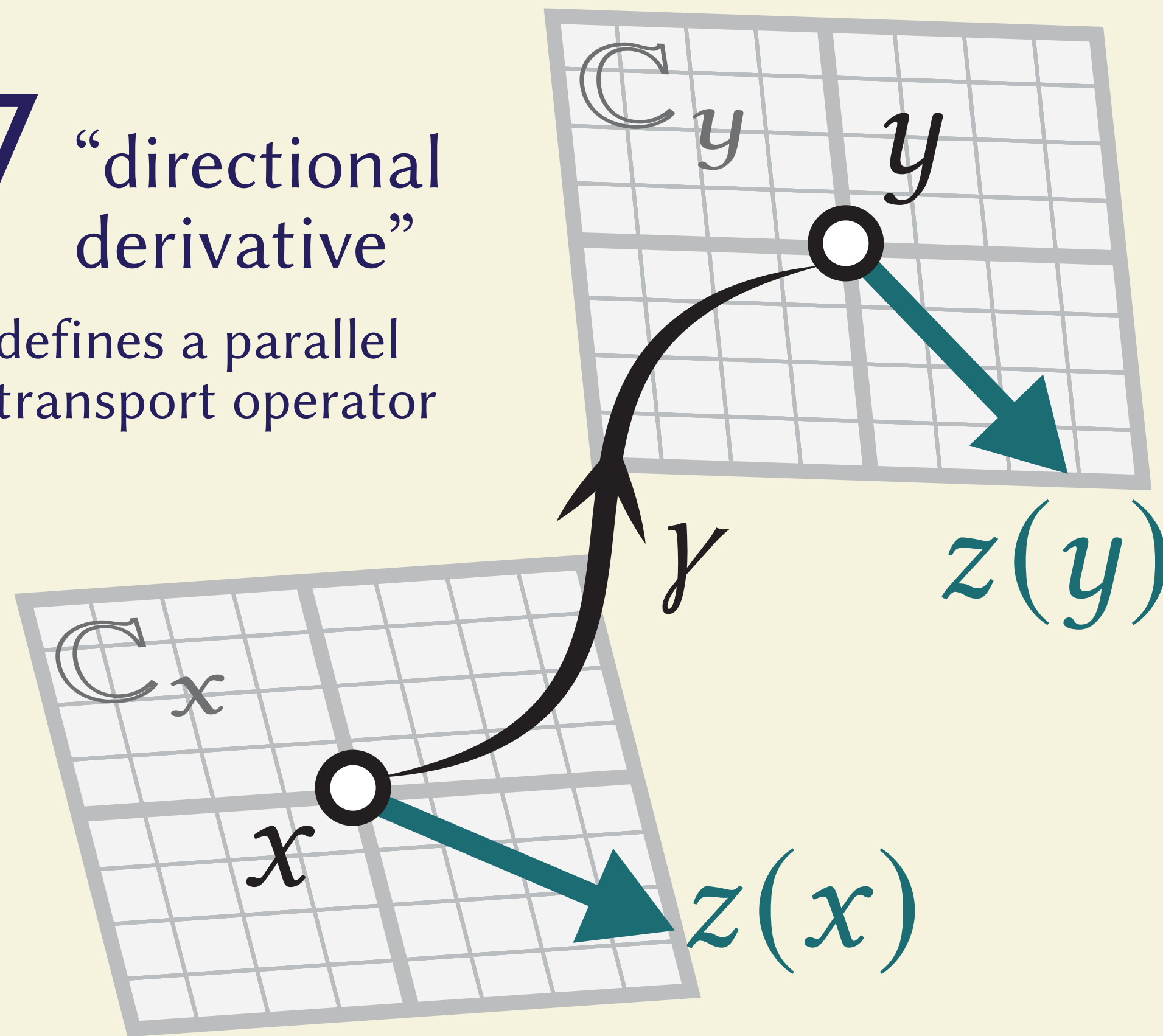
local structure: $M \times \mathbb{C}_x$
global structure may differ

Connections and Curvature



“directional derivative”

defines a parallel transport operator



“curvature 2-form”

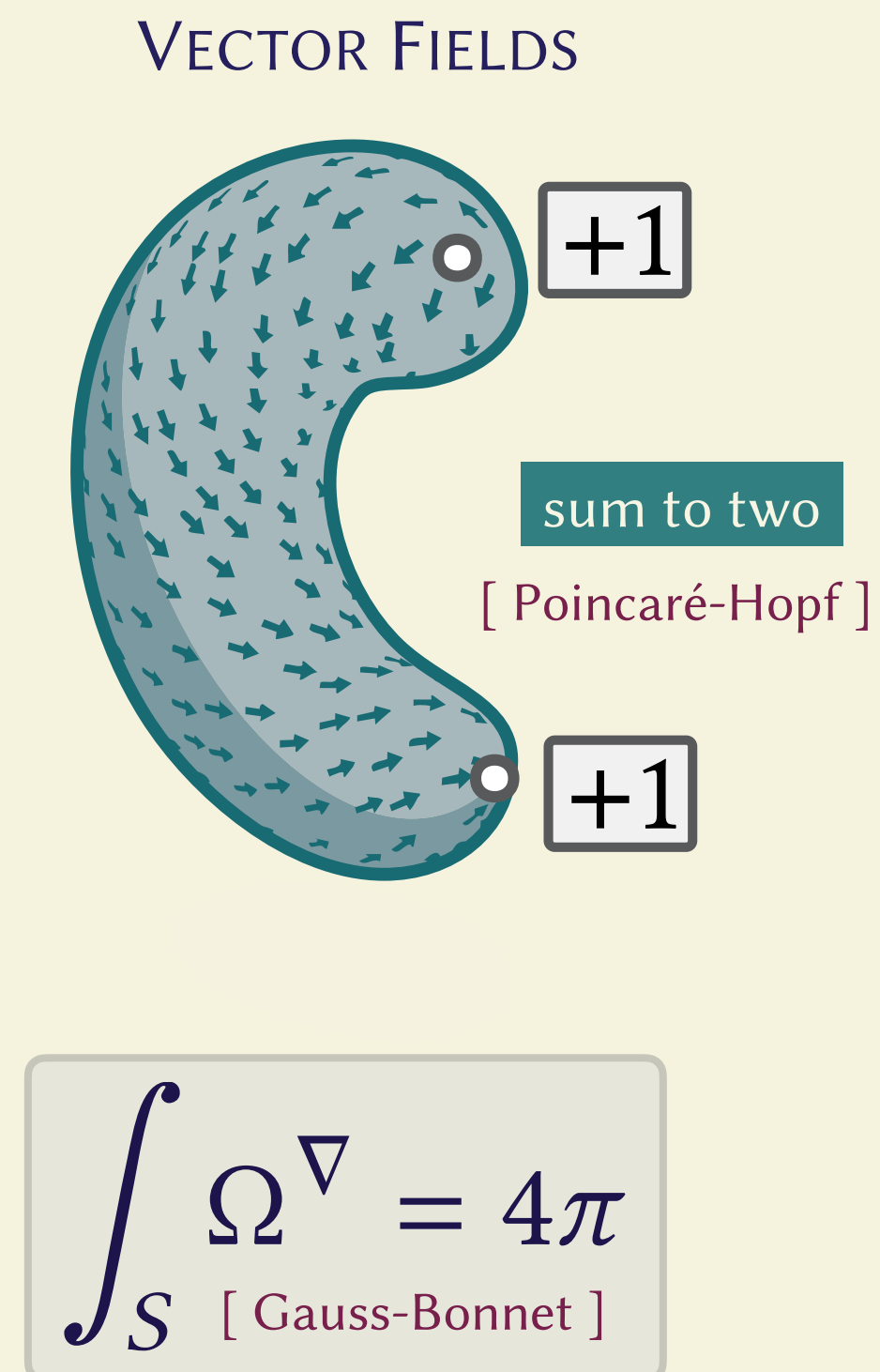
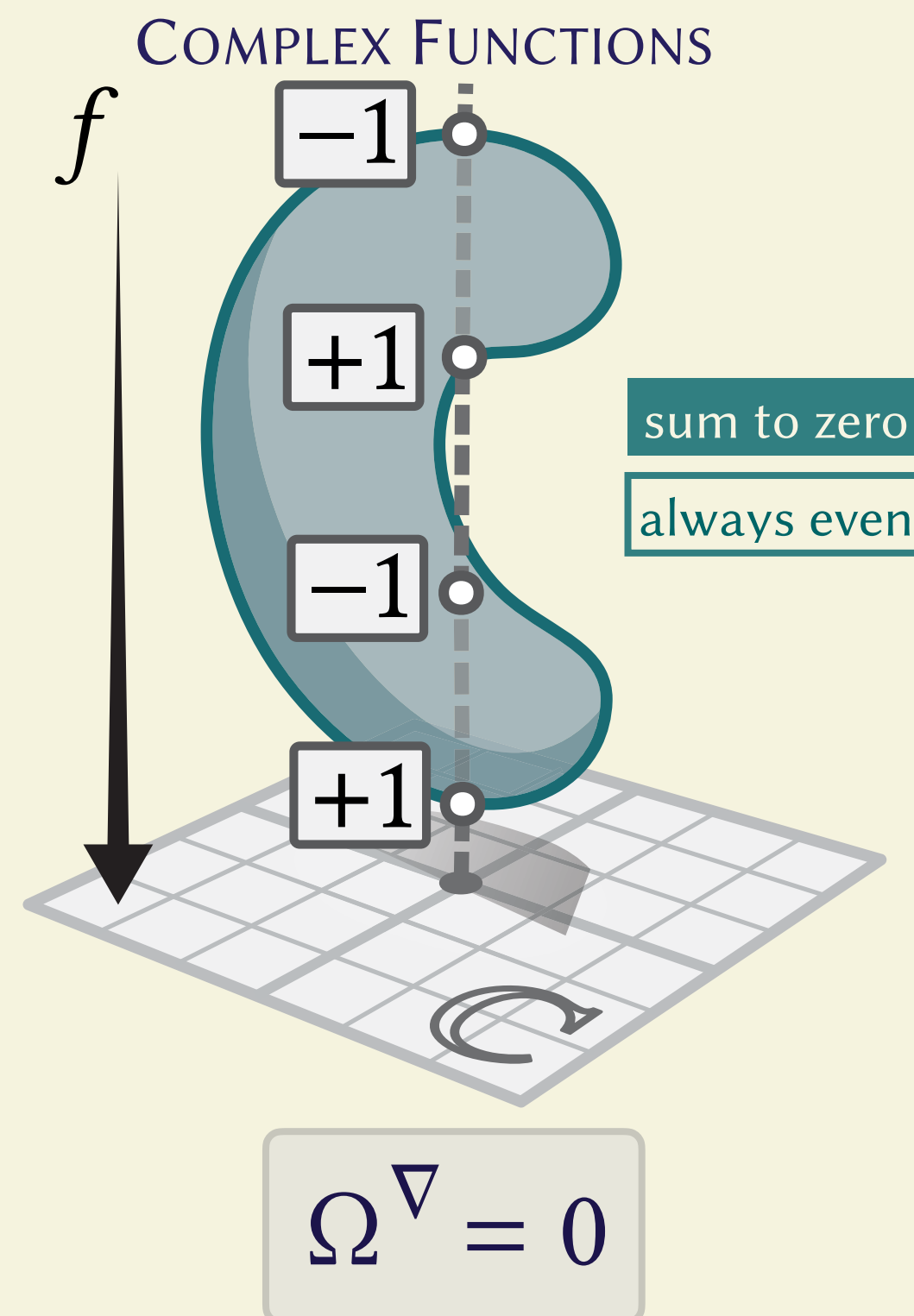
failure to close up
around small loops

$\left[\frac{1}{2\pi}\Omega^\nabla\right]$ is the first Chern class, which encodes the global structure [Chern 1946]

The Curvature 2-Form

for a complex line bundle on a surface S

$$\text{sum of zeros of a section} = \frac{1}{2\pi} \int_S \Omega^\nabla$$



Ω^∇ “curvature 2-form”
failure to close up
around small loops

$\left[\frac{1}{2\pi}\Omega^\nabla\right]$ is the first Chern class, which
encodes the global structure [Chern 1946]

for a complex line bundle on a
general manifold

THEOREM:

The first Chern class $\left[\frac{1}{2\pi}\Omega^\nabla\right]$ is equal to the
Euler class of the complex line bundle, *i.e.* the
Poincaré dual to the zero set of any generic
section.

for any closed 2-dimensional submanifold S

$$\frac{1}{2\pi} \int_S \Omega^\nabla = \# \text{ oriented intersections between } S \text{ and the zero set}$$

Discrete Complex Line Bundles

[Knöppel & Pinkall 2016]

on a d -dimensional simplicial complex $M = (V, E, F, \dots)$

1. A copy $\mathbb{C}_i$ of the complex plane for each vertex $i \in V$
2. A parallel transport map $r_{ij} \in \mathbb{C}$ for each edge $ij \in E$
3. A curvature $\Omega_{ijk} \in \mathbb{R}$ for each face $ijk \in F$

COMPATIBILITY CONDITION

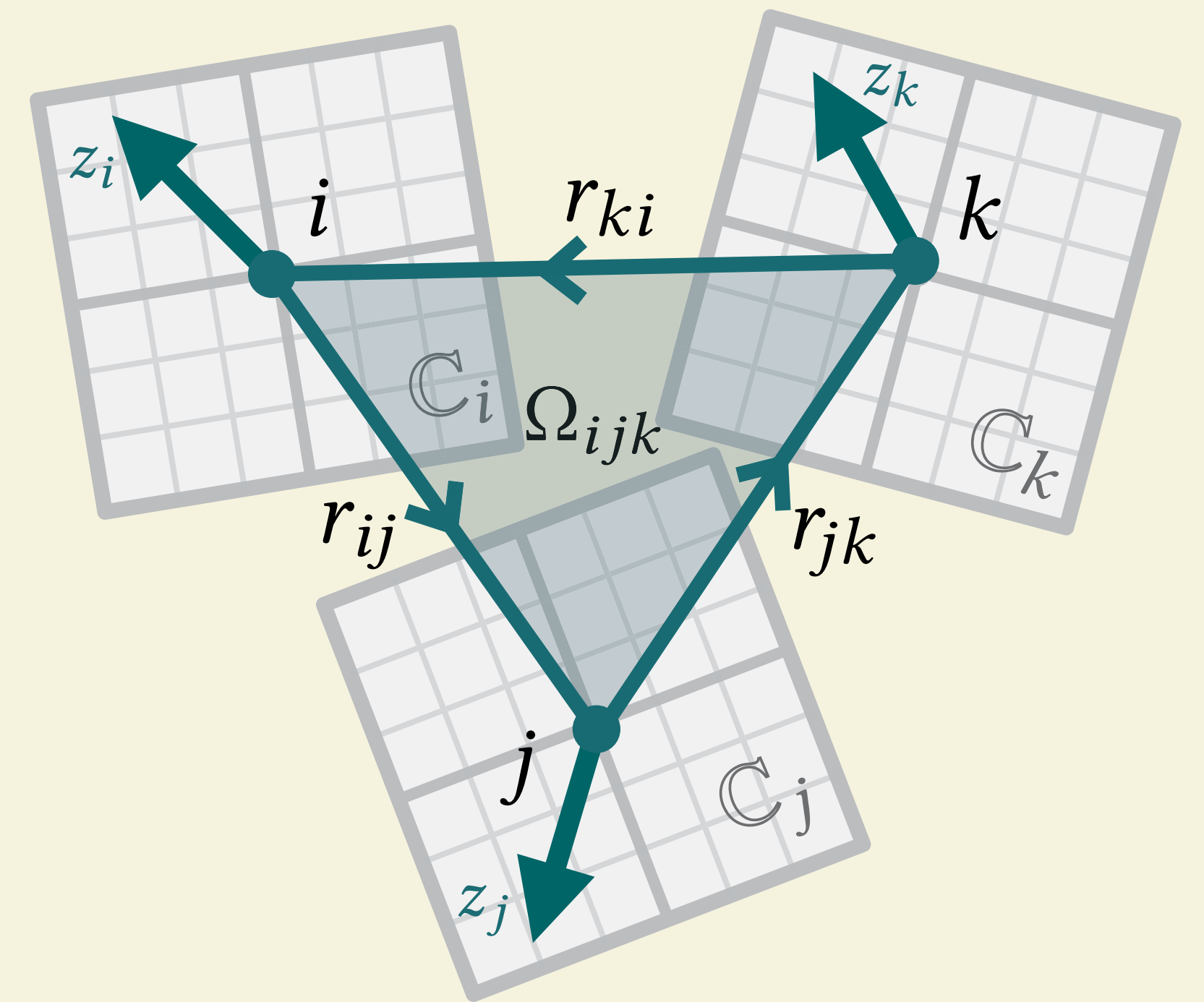
$$r_{ij} r_{jk} r_{ki} = \exp(i \Omega_{ijk})$$

(Ω_{ijk} measures rotation around face ijk)

A *discrete section* is a choice of complex number

$$z_i \in \mathbb{C}_i$$

for each $i \in V$



Discrete Complex Line Bundles

[Knöppel & Pinkall 2016]

THEOREM [Knöppel & Pinkall 2016]:

Fix a base simplicial complex M .
If $\Omega \in \Omega^2(M, \mathbb{R})$ is integral, then
there exists a discrete complex
line bundle with curvature Ω

(EXISTENCE)

COMPATIBILITY CONDITION

$$r_{ij} r_{jk} r_{ki} = e^{2\pi i \Omega_{ijk}}$$

(Ω_{ijk} measures rotation around face ijk)

A *discrete section* is a
choice of complex number

$$z_i \in \mathbb{C}_i$$

for each $i \in V$

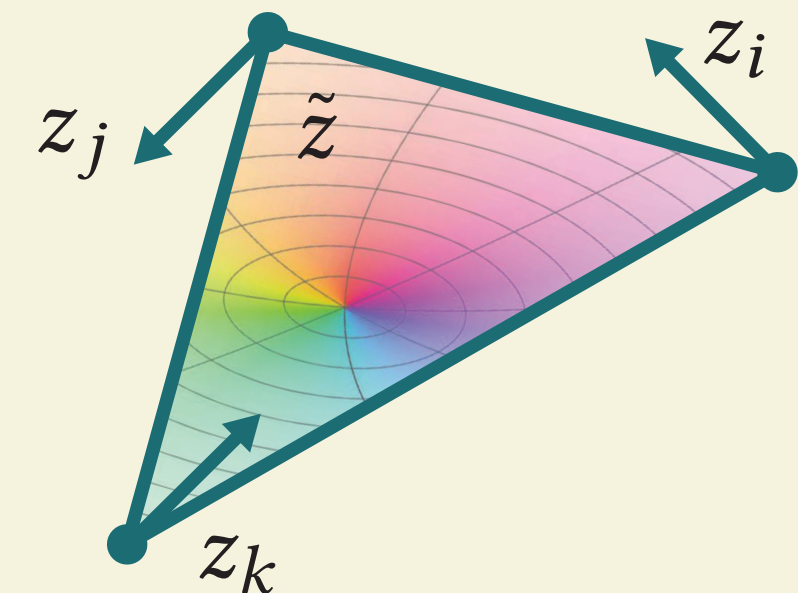
THEOREM [Knöppel & Pinkall 2016]:

Let $L \rightarrow M$ be a discrete complex line
bundle with curvature Ω . The Poincaré dual
of the zero set of a discrete section ψ is
given by the 2-form $\text{ind}^\psi = \frac{1}{2\pi} (\Omega + d\xi^\psi)$,
which is cohomologous to the curvature
form $\frac{1}{2\pi}[\Omega]$.

(STRUCTURE OF ZERO SETS)

THEOREM [Knöppel & Pinkall 2016]:

A discrete section ψ of a discrete
complex line bundle $L \rightarrow M$ can be
identified with a piecewise-linear
section $\tilde{\psi}$ of an associated smooth
complex line bundle $\tilde{L} \rightarrow M$.



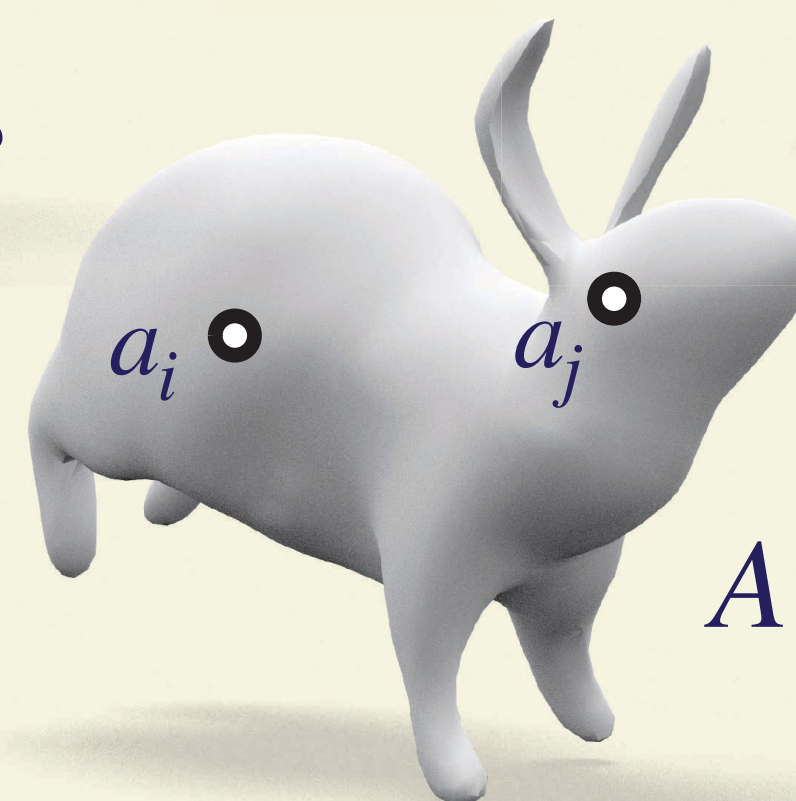
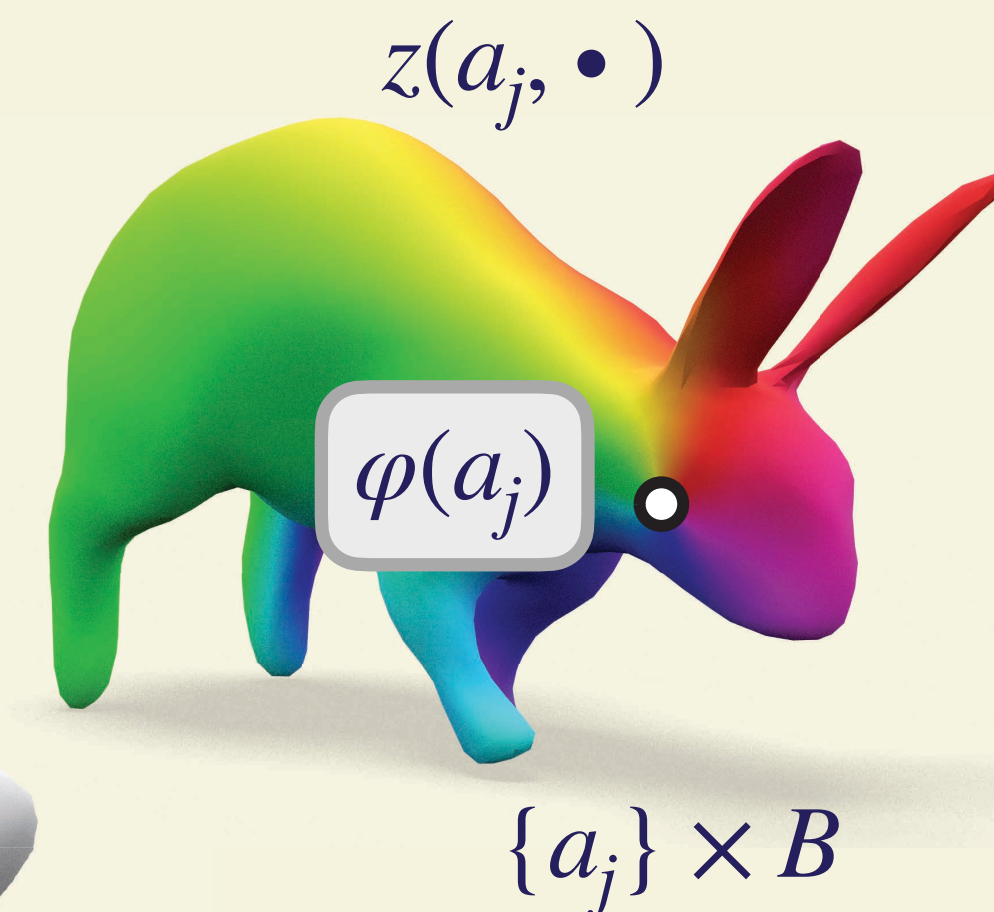
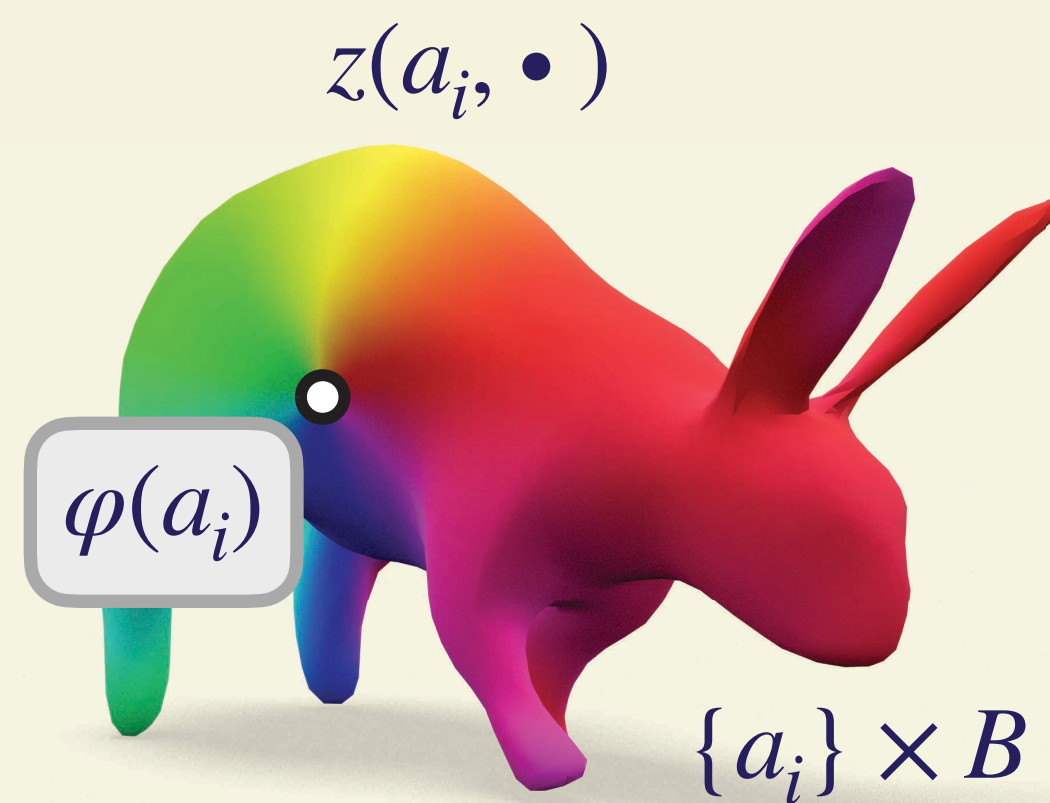
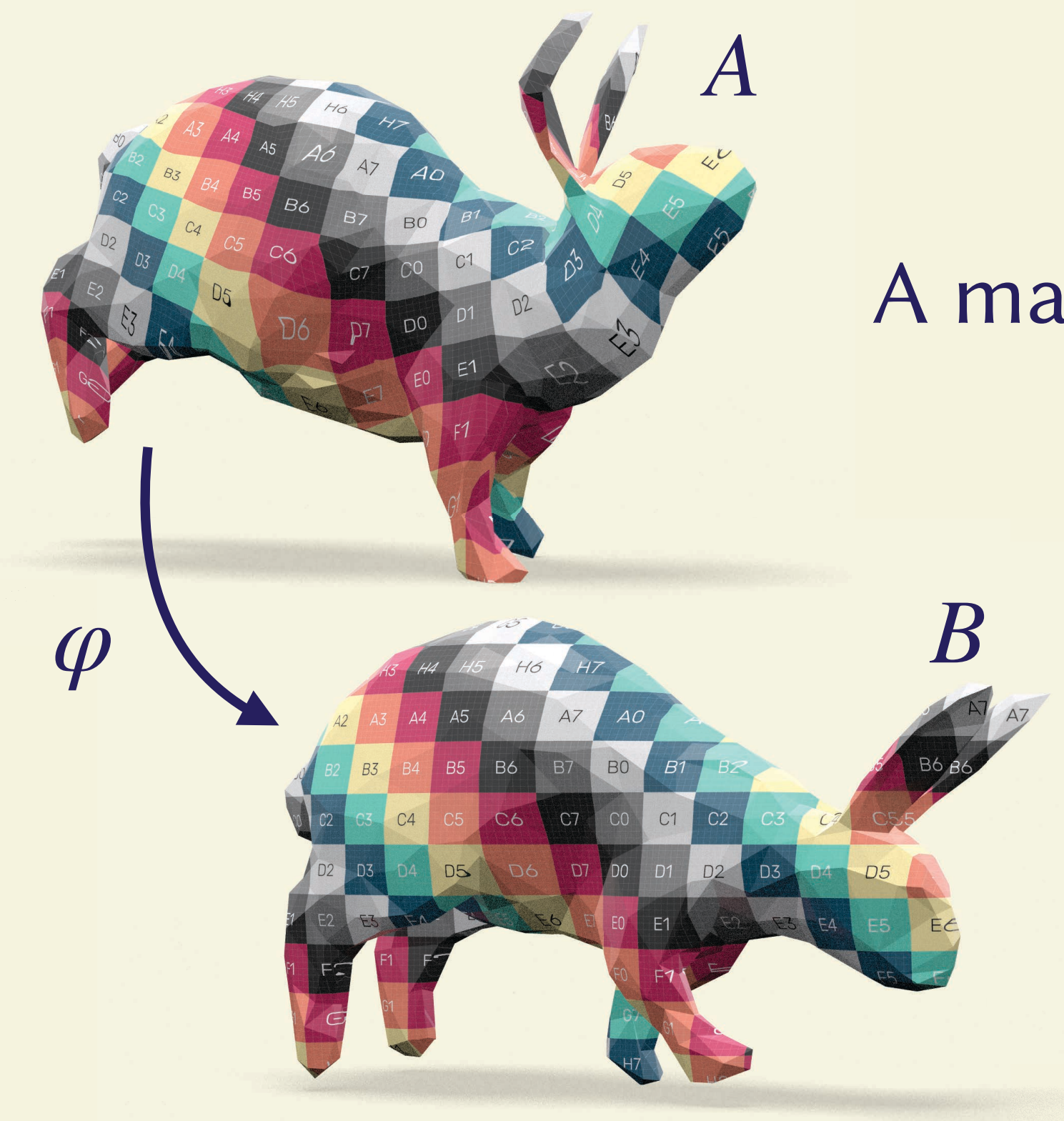
(INTERPOLATION)

Representing a Map as a Section

If we know z , we can evaluate $\varphi(a)$ for any point $a \in A$

Idea: Think of a map as the zero set of a section on $A \times B$

A map $\varphi : A \rightarrow B \iff$ A section z on $A \times B$ with $z(a, b) = 0$ if $\varphi(a) = b$



Extra Information: Edge Crossings

structure preserving discrete
complex line bundles $\Rightarrow$

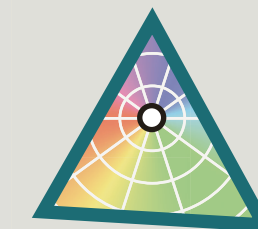
implicit representation correctly
encodes combinatorics

We can read off the common
refinement directly from the section z

Faces in $A \times B$

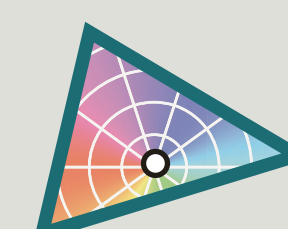
$$v_A \times f_B$$

where does
 v_A land in f_B ?



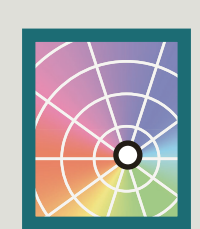
$$f_A \times v_B$$

where does
 v_B land in f_A ?



$$e_A \times e_B$$

where does e_A
intersect e_B ?



only uses complex values z on $A \times B$

How do we build the complex line bundle?

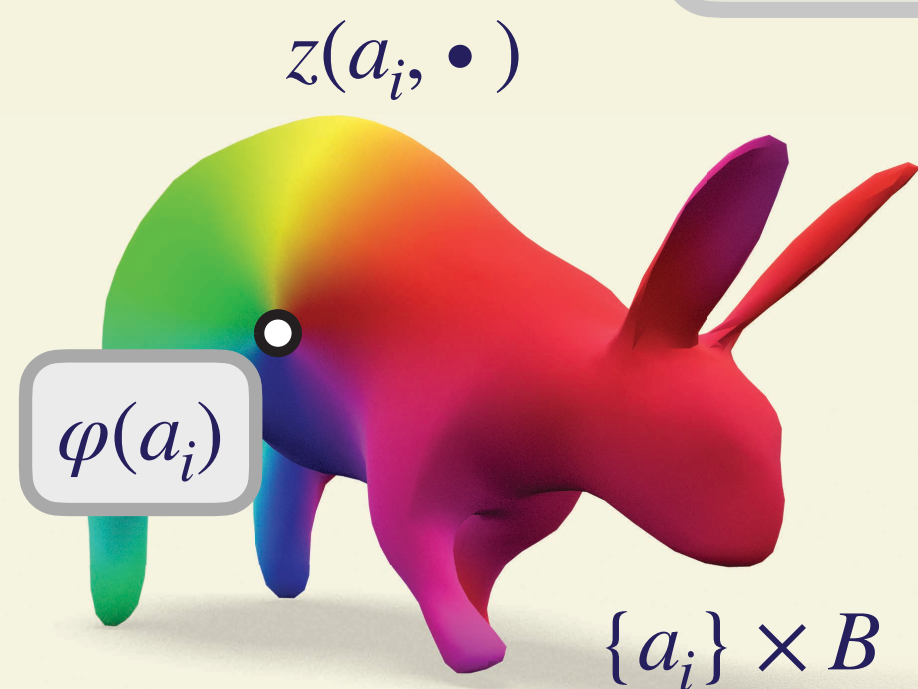
Need to pick:

- A parallel transport map $r_{ij} \in \mathbb{C}$ for each edge $ij \in E$
- A curvature $\Omega_{ijk} \in \mathbb{R}$ for each face $ijk \in F$

Existence: [Knöppel & Pinkall 2016]

Easy construction:

- Build connections on A, B with total curvature 2π
solve via *trivial connections* [Crane et al. 2010]
- Take *product connection* on $A \times B$



pick a connection
with total curvature
 2π on each slice

want one zero per slice

$$\begin{aligned} \# \text{ oriented intersections between } S \text{ and the zero set of a section} &= \frac{1}{2\pi} \int_S \Omega^\nabla \\ \text{first Chern class } \left[\frac{1}{2\pi} \Omega^\nabla \right] &\text{ is dual to diagonal class} \\ [\Delta] &= [\text{pt}_A \times B + A \times \text{pt}_B] \end{aligned}$$

How do we build the complex line bundle?

Need to pick:

- A parallel transport map $r_{ij} \in \mathbb{C}$ for each edge $ij \in E$
- A curvature $\Omega_{ijk} \in \mathbb{R}$ for each face $ijk \in F$

guarantees that map will lie in correct topological class

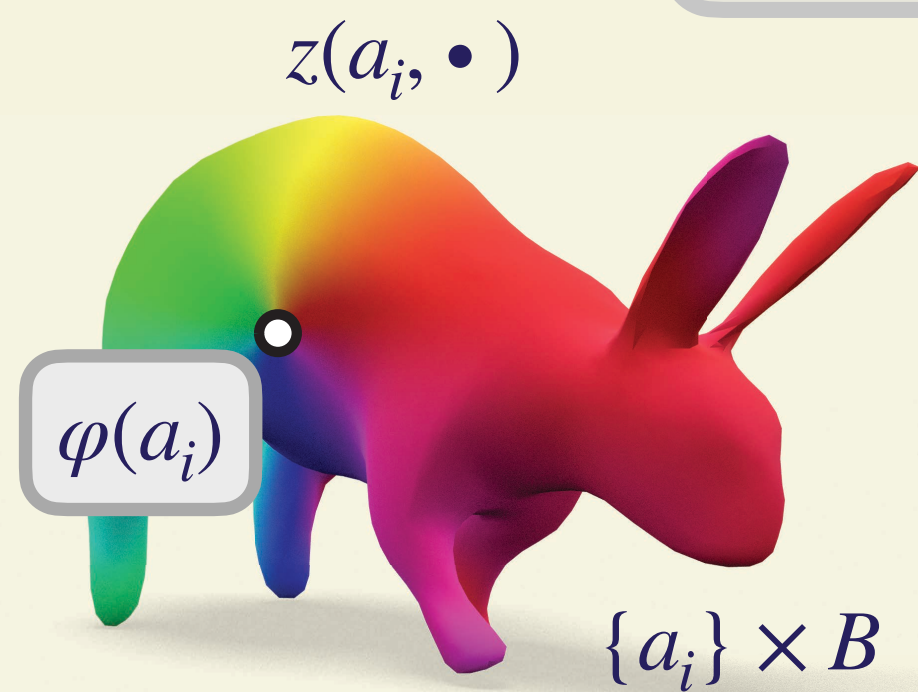
Existence: [Knöppel & Pinkall 2016]

Easy construction:

- Build connections on A, B with total curvature 2π
solve via *trivial connections* [Crane et al. 2010]
- Take *product connection* on $A \times B$

Uniqueness?

- Determined up to gauge transformation by curvature 2-form — in practice, $\frac{1}{2}$ Gaussian curvature



want one zero per slice

pick a connection with total curvature 2π on each slice

first Chern class $\left[\frac{1}{2\pi} \Omega^\nabla \right]$ is dual to diagonal class

$$[\Delta] = [\text{pt}_A \times B + A \times \text{pt}_B]$$

oriented intersections between S and the zero set of a section

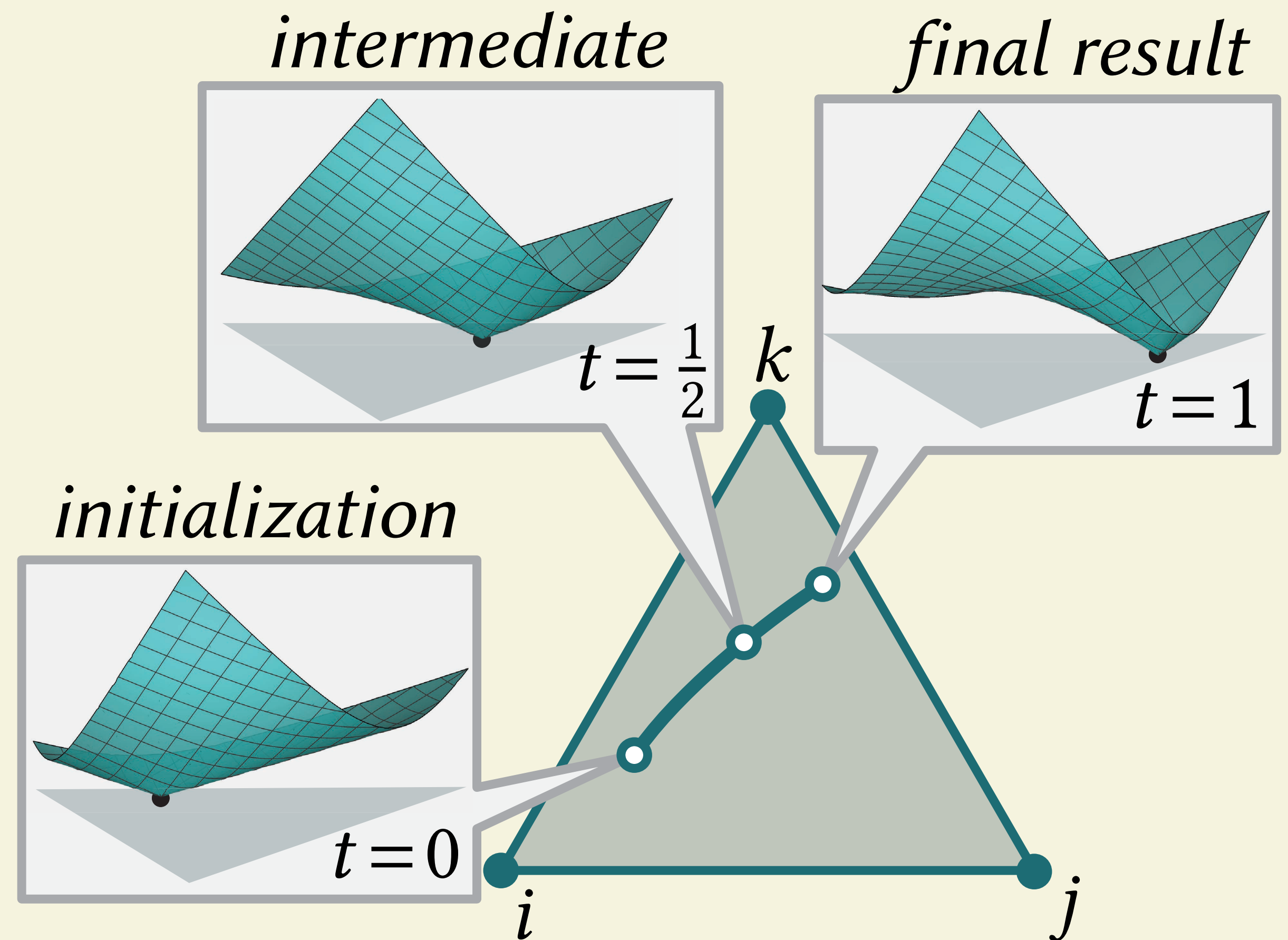
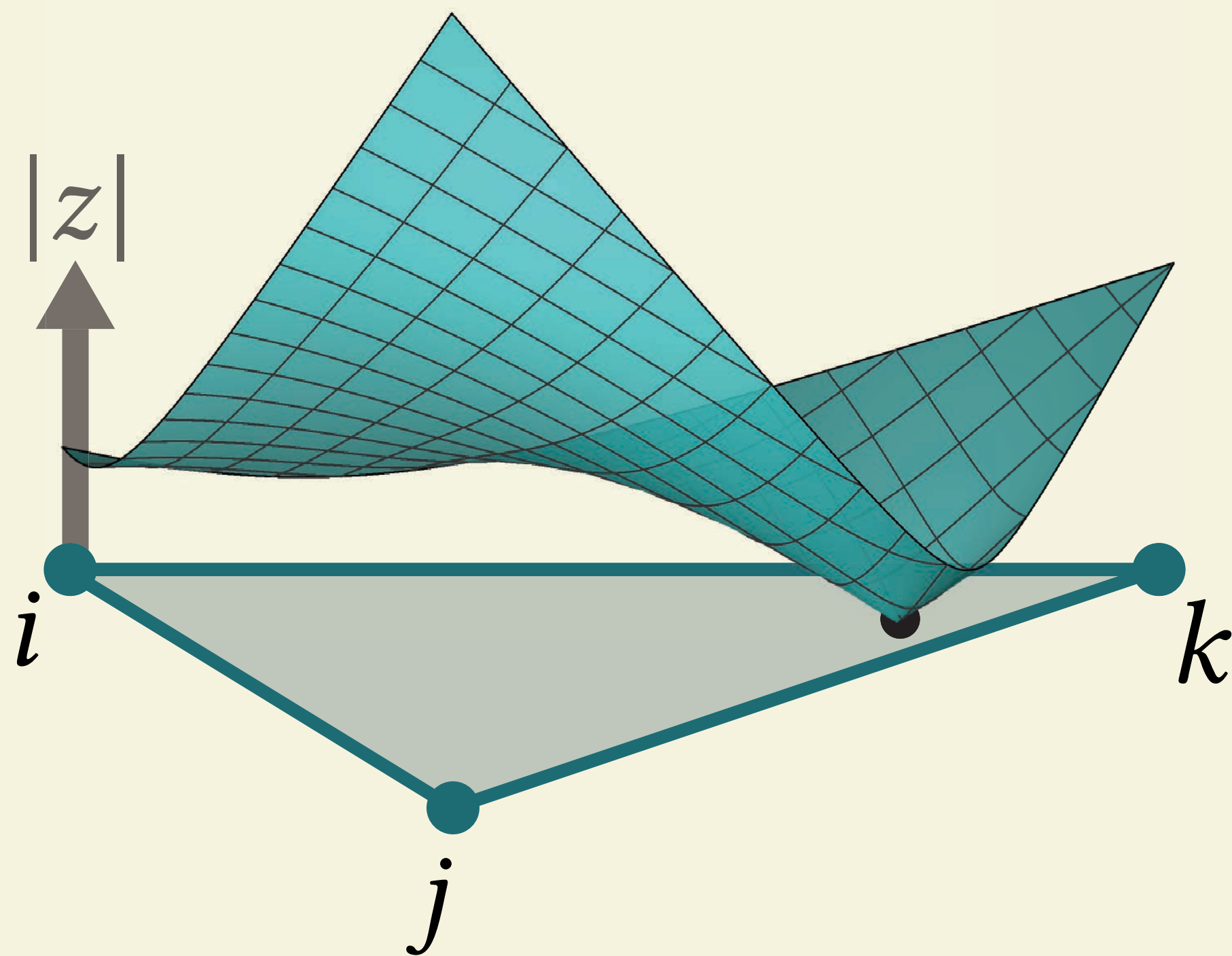
$$= \frac{1}{2\pi} \int_S \Omega^\nabla$$

Finding Singularities

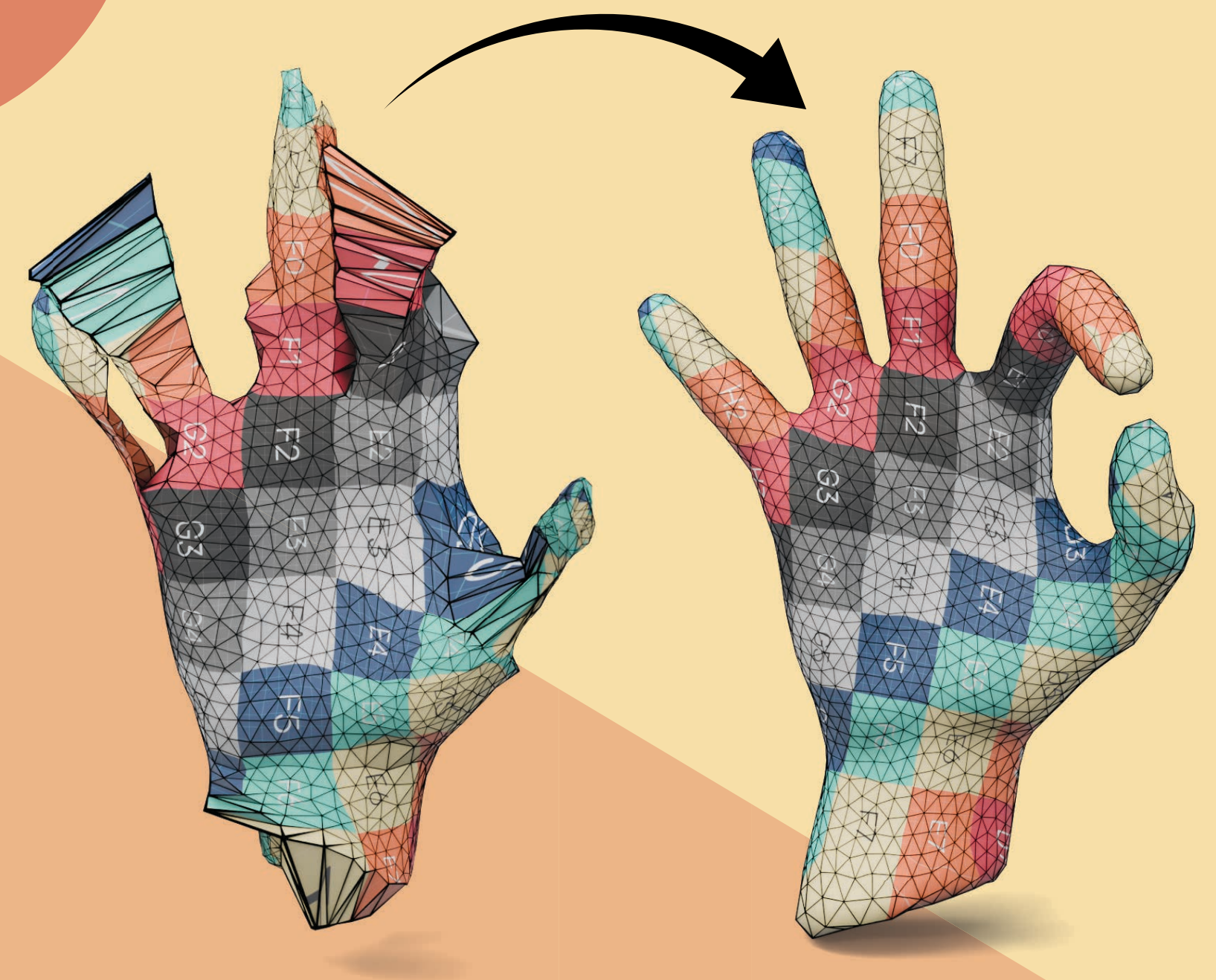
$$(1 - b_j - b_k)|z_i| + b_j|z_j|e^{l(b_k\Omega + \omega_{ij})} + b_k|z_k|e^{-l(b_j\Omega + \omega_{ki})} = 0.$$

$$\omega_{ij} := \arg\left(\frac{z_j}{r_{ij}z_i}\right) \in [-\pi, \pi).$$

[Knöppel & Pinkall 2016]



2 Map Optimization



The Ginzburg-Landau Energy

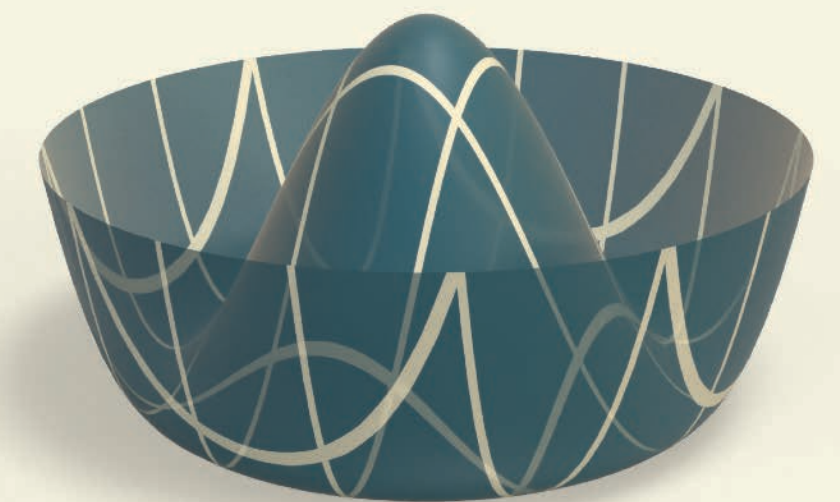
$$\mathcal{E}_\varepsilon^{\mathcal{GL}}(z) := \int_A \int_B \frac{1}{2} |\nabla z(p)|^2 + \frac{1}{4\varepsilon^2} \left(|z(p)|^2 - 1 \right)^2 dA dB$$

Dirichlet energy

encourages z to change as little as possible

Circular well potential

encourages z to have unit norm everywhere



At a minimizer:

gets very large
near zeros

z has unit norm,
except near zeros

make zero set as small as possible

minimizers provide minimal surfaces on flat bundles over open subsets of $\mathbb{R}^n$ [Lin and Riviere 1999]
for nontrivial bundles over simply-connected domains, we have varifold convergence of Jacobians [Canevari et al. 2023]

Minimizing the Ginzburg-Landau Energy

discrete section: $Z \in \mathbb{C}^{V_A \times V_B}$

$$\mathcal{E}_\varepsilon^{\mathcal{GL}}(Z) = \frac{1}{2} \text{Tr} \left[\bar{Z}^\top L_A^\nabla Z (M_B^\nabla)^\top \right] + \frac{1}{2} \text{Tr} \left[\bar{Z}^\top M_A^\nabla Z (L_B^\nabla)^\top \right] + \frac{1}{4\varepsilon^2} \text{Tr} \left[U^\top M_A U M_B^\top \right]$$

some matrix expression of Z $\left(U_{i,j} = |Z_{i,j}|^2 - 1 \right)$

$L_A^\nabla, L_B^\nabla, M_A^\nabla, M_B^\nabla, M_A, M_B$ *connection Laplacian, mass matrices* — fixed matrices depending on A and B
[Knöppel & Pinkall 2016]

optimize with off-the-shelf first order optimizer, e.g. L-BFGS [Liu & Nocedal 1989]

Landmark Points

singularity pinning [Aftalion et al. 2001]

CIRCULAR WELL POTENTIAL

$$\mathcal{W}(z) := \frac{1}{4} \int_A \int_B \left(|z(p)|^2 - 1 \right)^2 dA dB$$

“PINNED” CIRCULAR WELL POTENTIAL

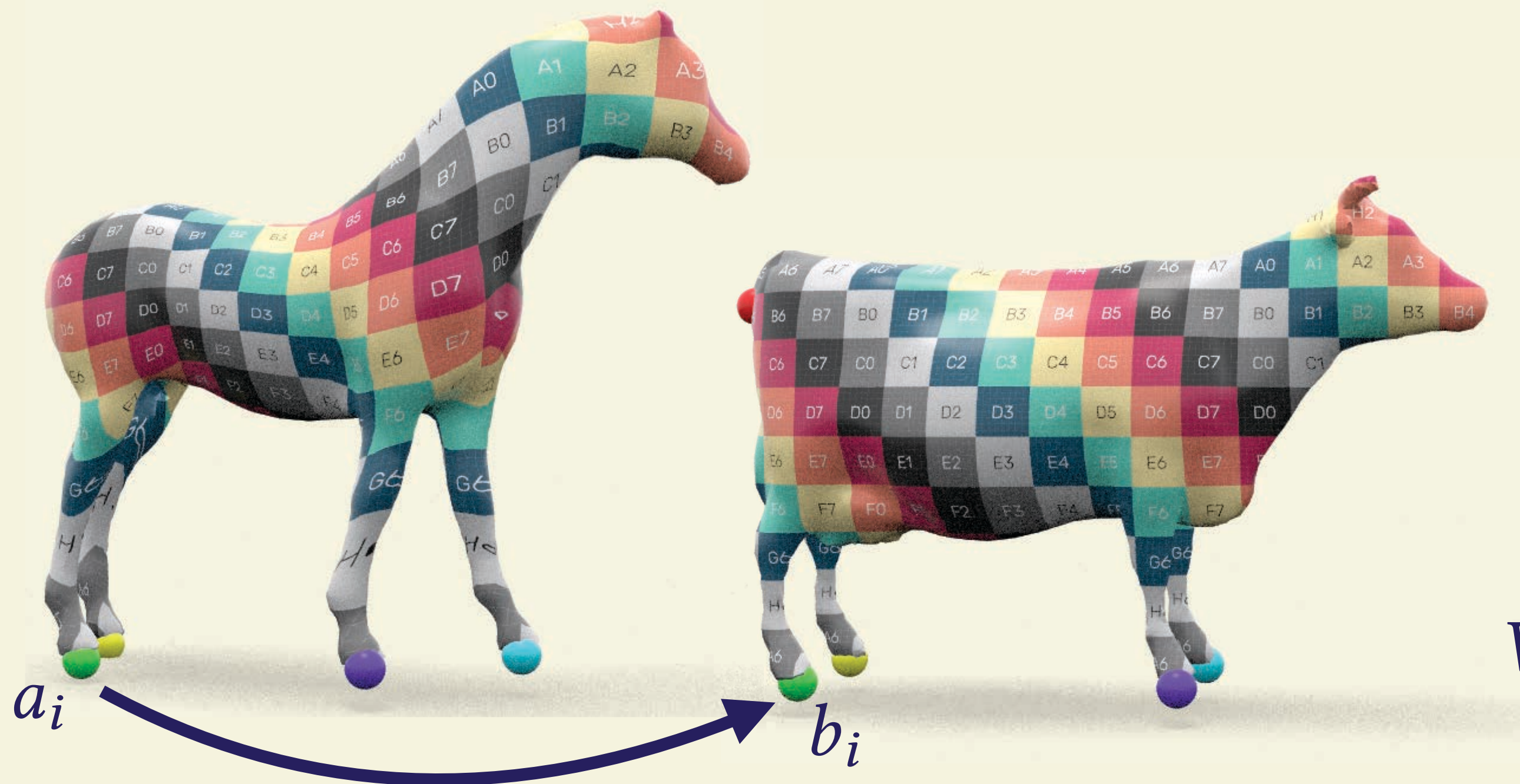
$$\mathcal{W}(z) := \frac{1}{4} \int_A \int_B \left(|z(p)|^2 - V(p) \right)^2 dA dB$$

$V(p)$ determines target magnitude

$$V(p) := 1 - \min_{\text{landmark pairs } (a_i, b_i)} \exp \left(- d_{A \times B} [p, (a_i, b_i)]^2 \right)$$

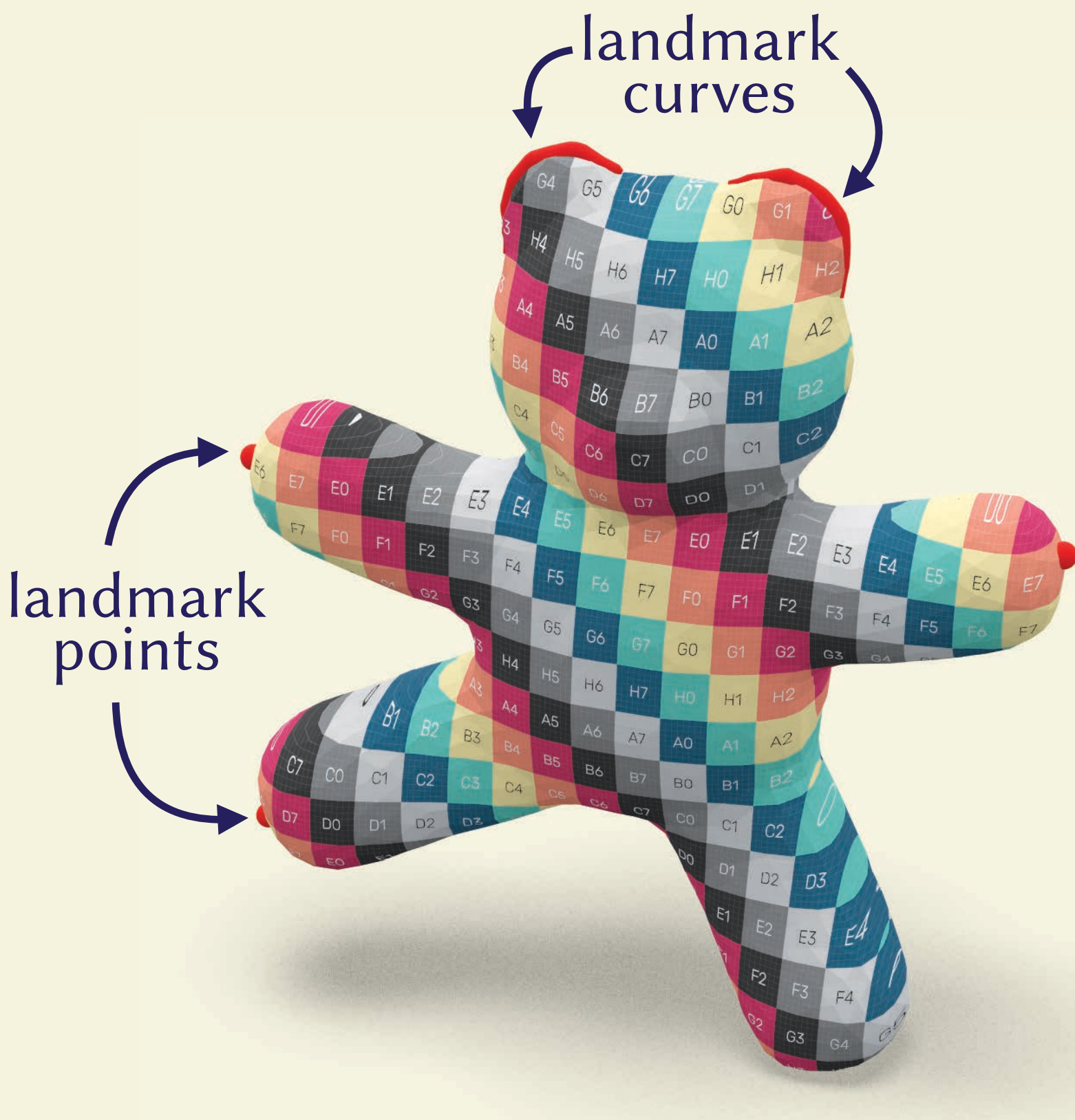
encourage zeros to form around $(a_i, b_i) \in A \times B$

soft penalty encouraging
 a_i to map to b_i

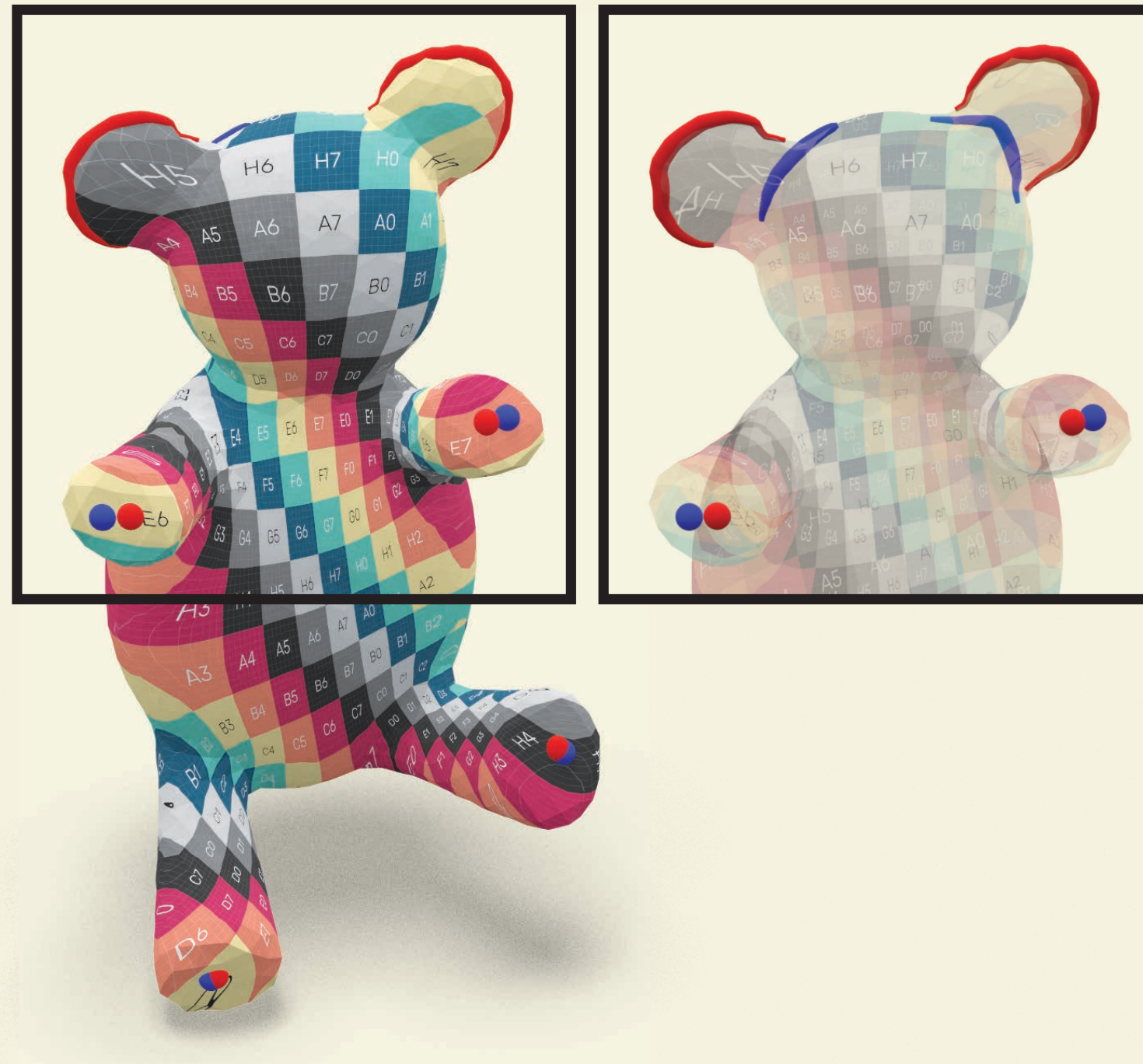


Landmark Curves

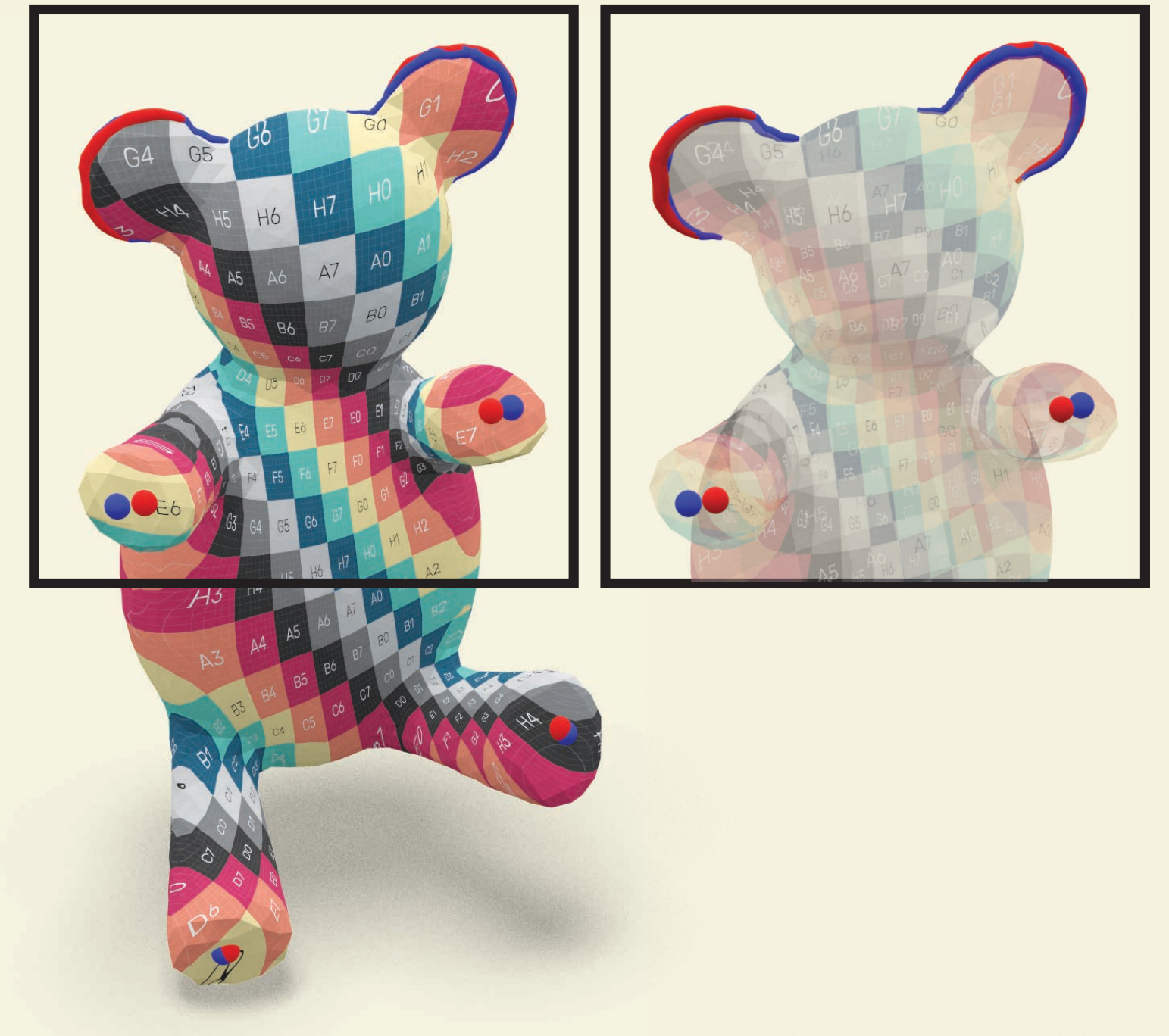
■ landmark
■ mapped landmark



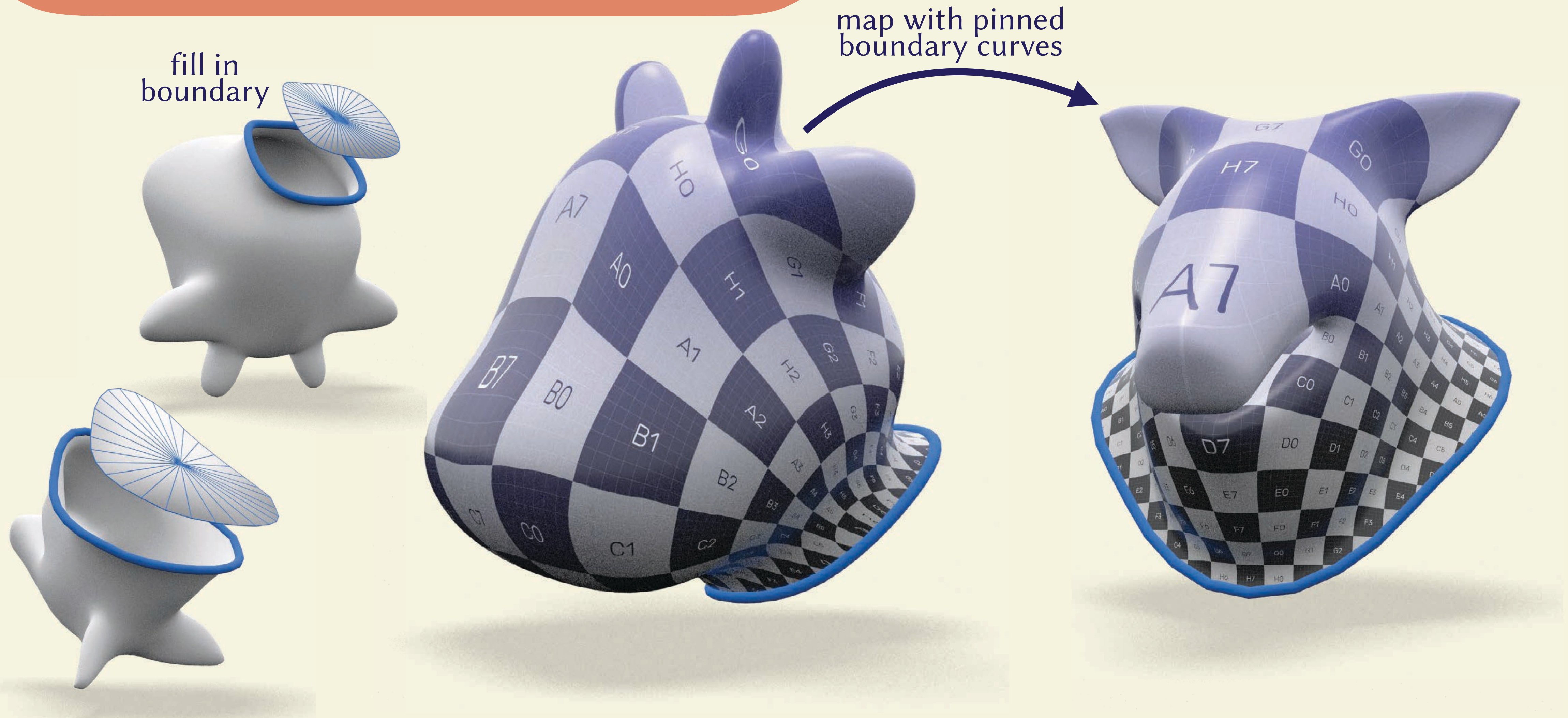
using landmark points only



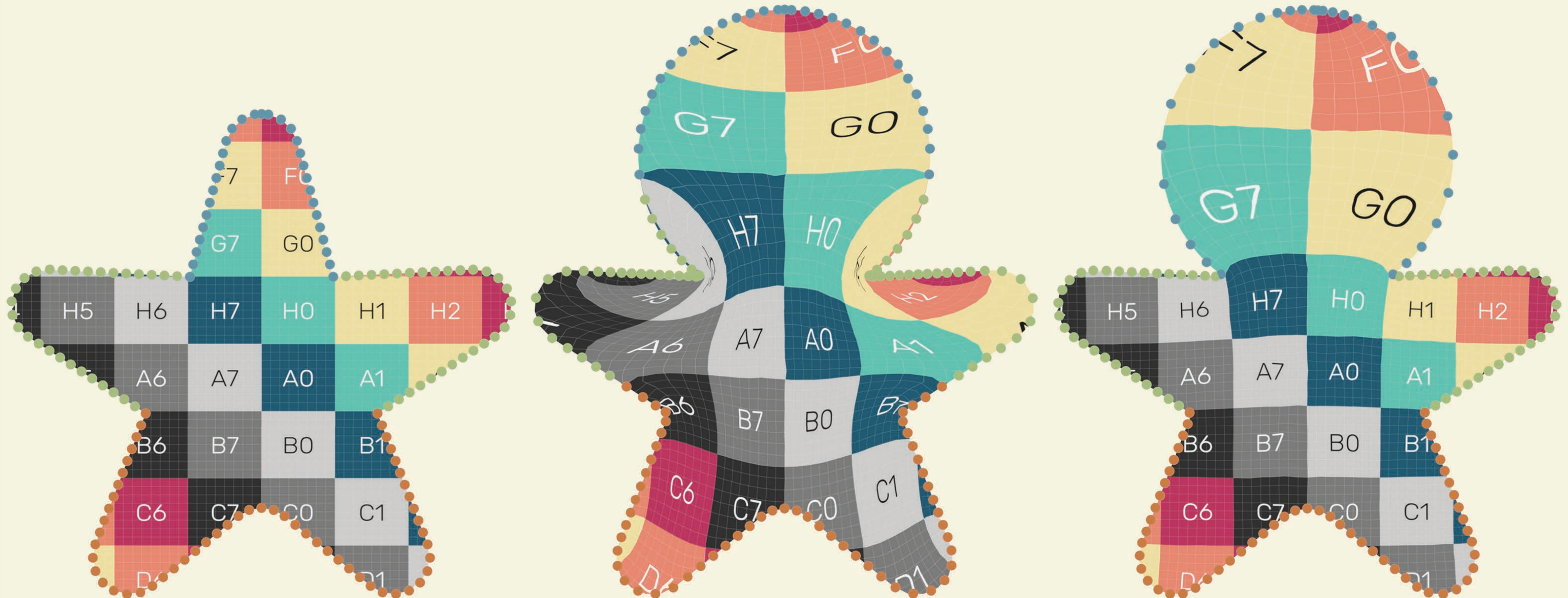
using landmark points & curves



Handling Boundaries



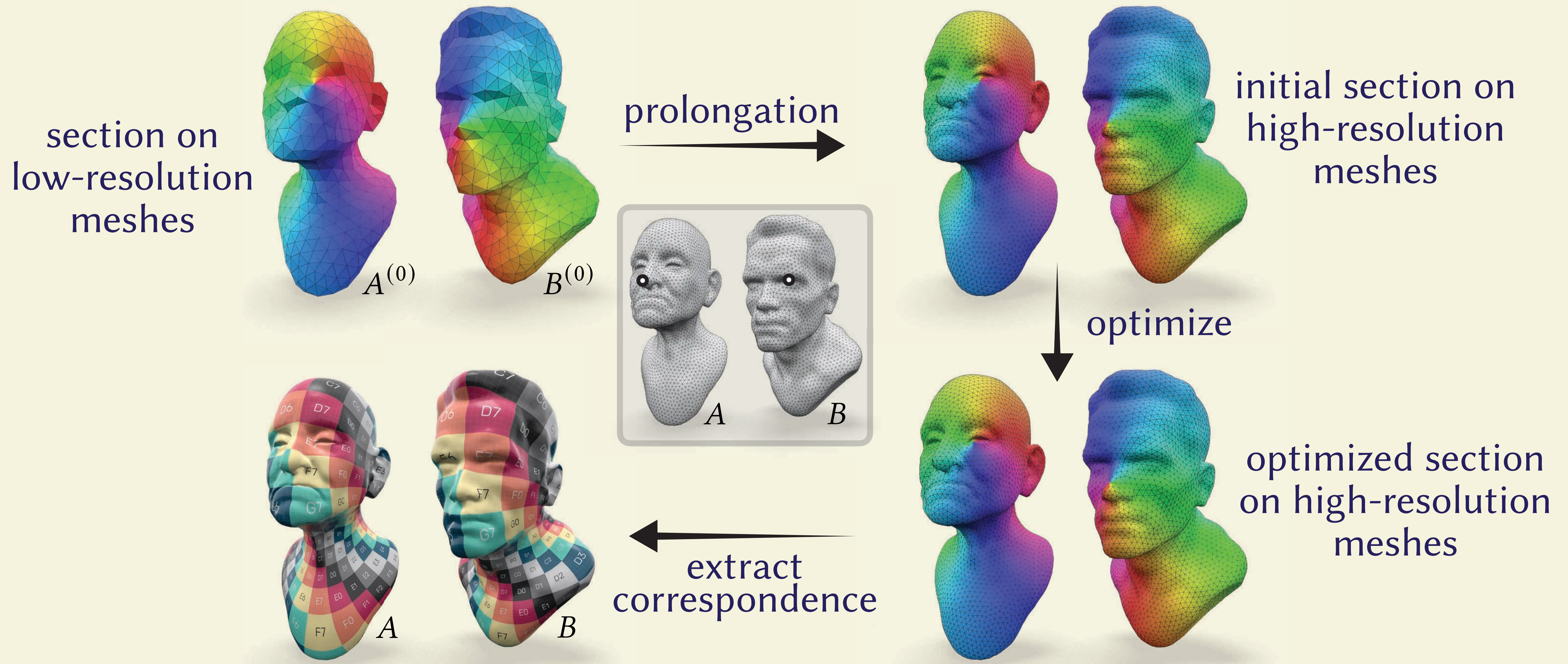
Constrains curve images, not pointwise values



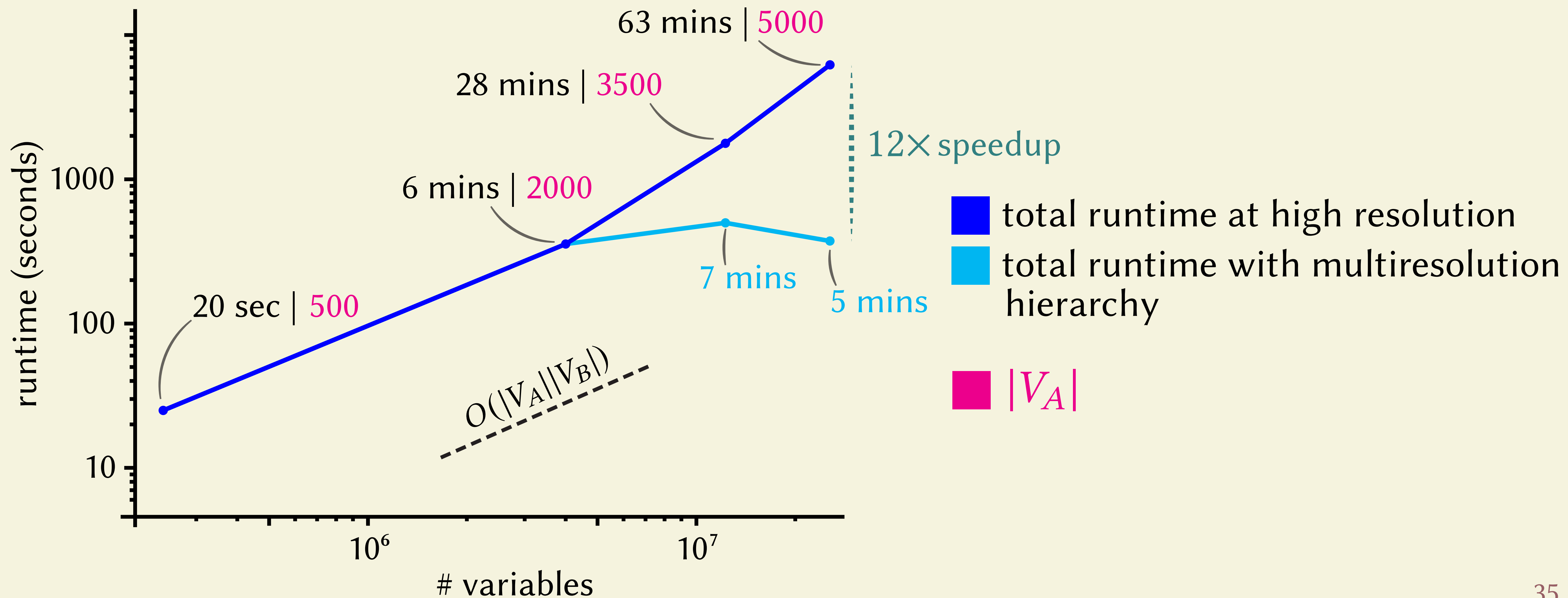
point constraints

curve constraints

Acceleration via Multiresolution Hierarchy



Runtime



Initialization

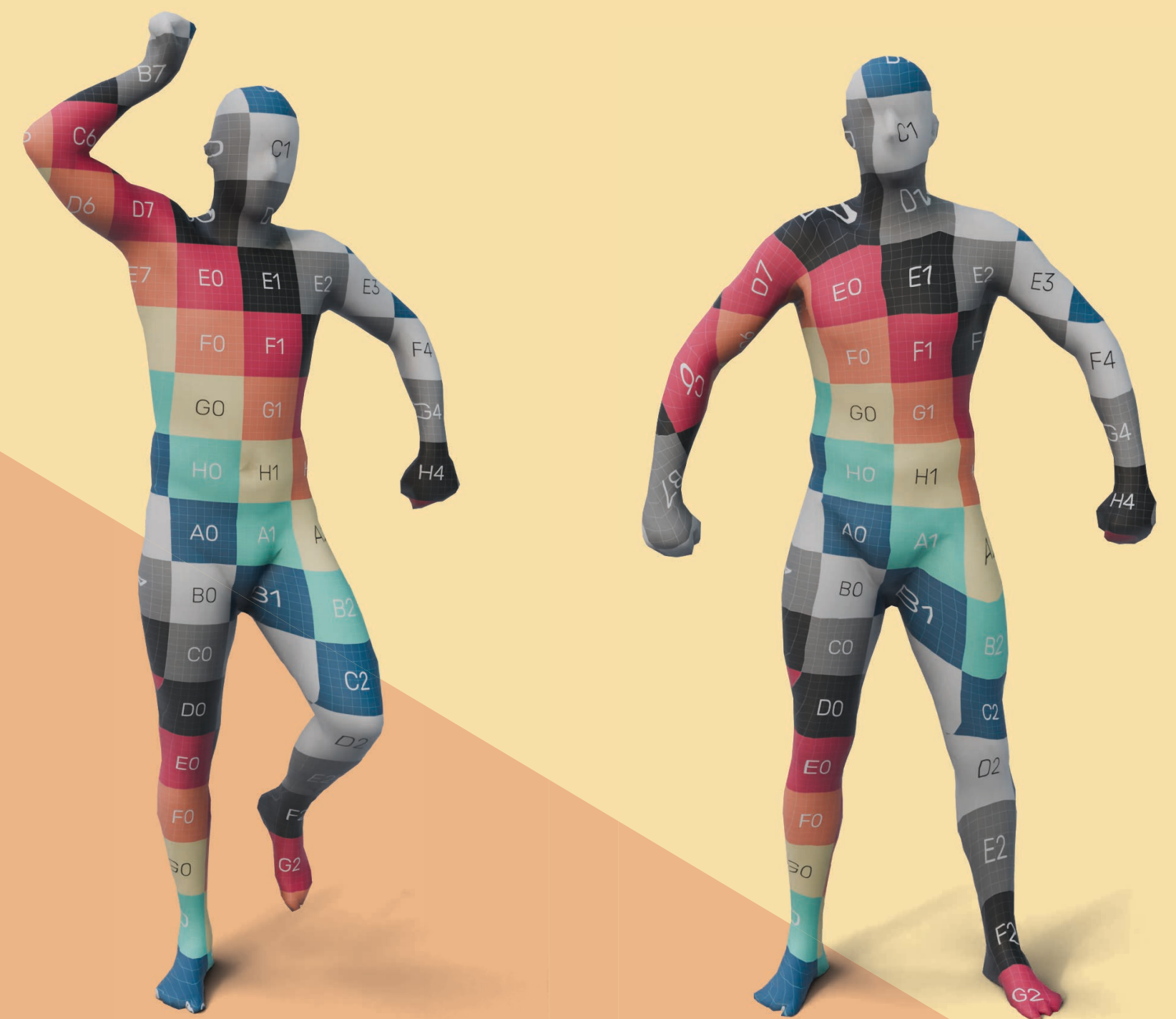
III. FIT IMPLICIT MAP



zeros of the smallest connection Laplacian eigenvector
seem to concentrate in regions of high curvature
[Knöppel et al. 2013; Weißmann et al. 2014]

construct an alternative connection on the product
space with curvature concentrated along initial map

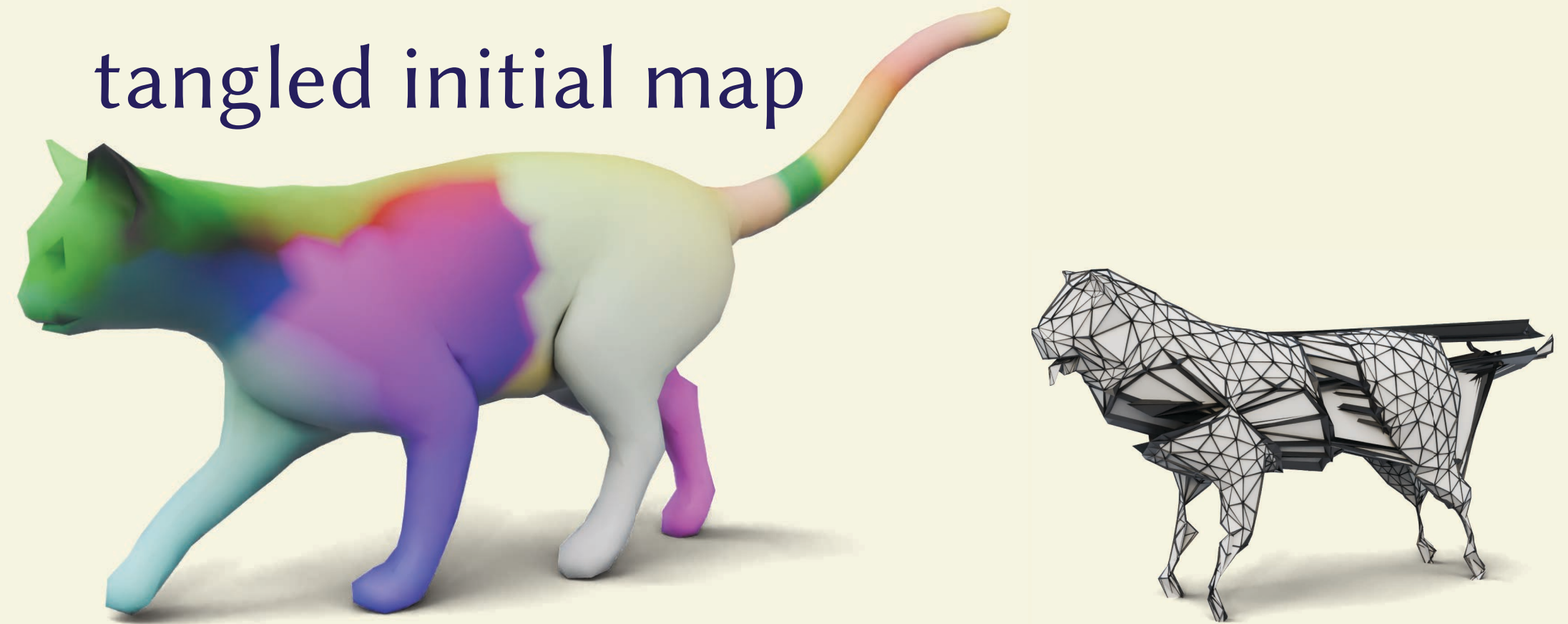
Examples and Comparisons



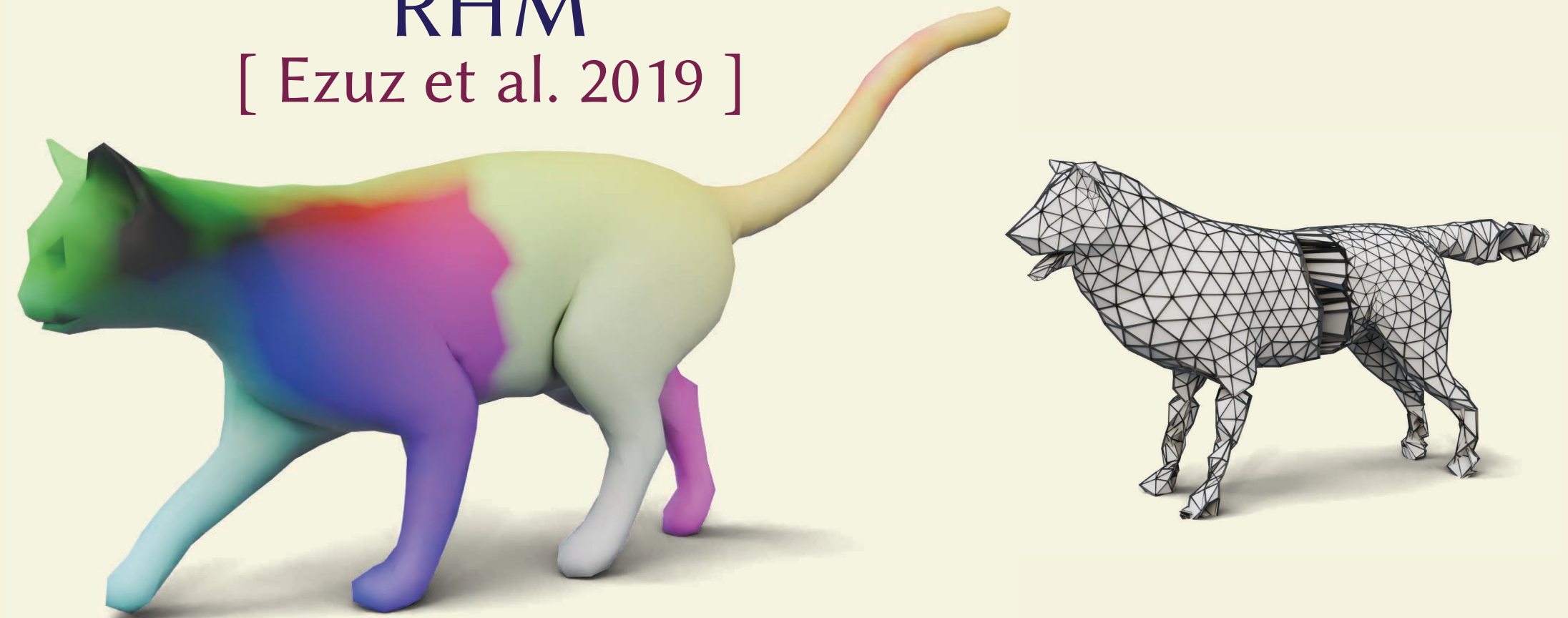
Map Untangling

Fixing Flips

tangled initial map



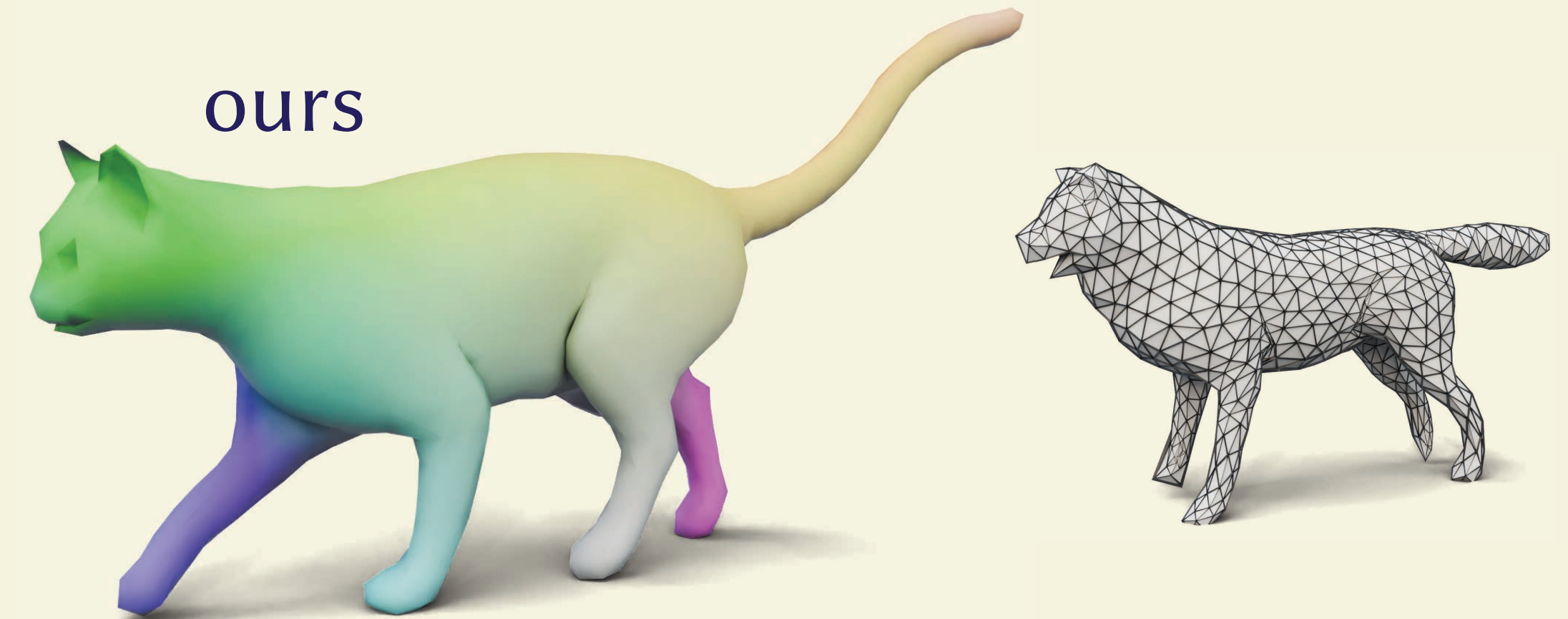
RHM
[Ezuz et al. 2019]



source



ours



Robustness to Initialization

Experiment from [Schmidt et al. 2020]

worse initialization

source mesh



RHM
[Ezuz et al. 2019]



ISM
[Schmidt et al. 2020]



Ours



target mesh



worse initialization

Map Distortion



source mesh



map
[ours]

distortion
(area)



distortion
(symmetric Dirichlet)



our simpler optimization
yields lower distortion in
both senses

distortion
(area)

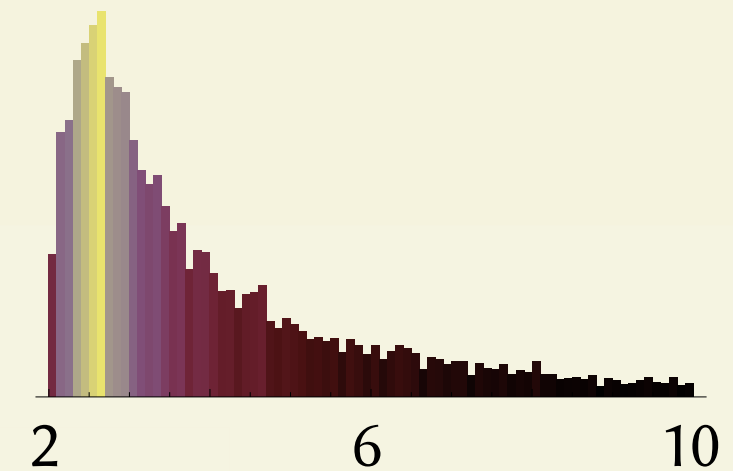
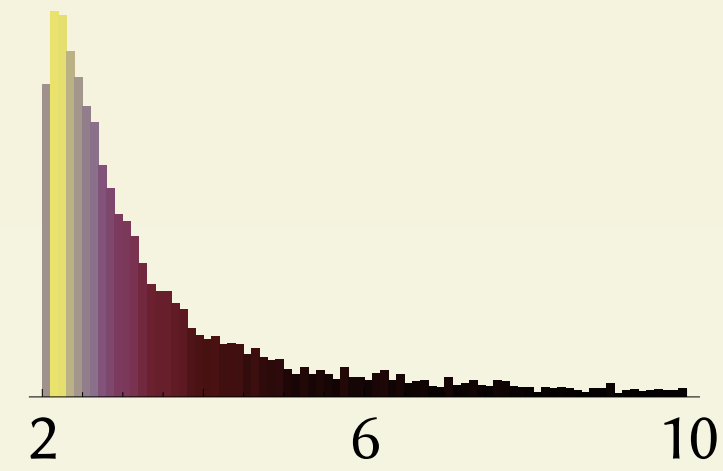


distortion
(symmetric Dirichlet)



map
[Schmidt et al. 2020]

Map Distortion On Non-isometric Shapes



source mesh



distortion (symmetric Dirichlet)

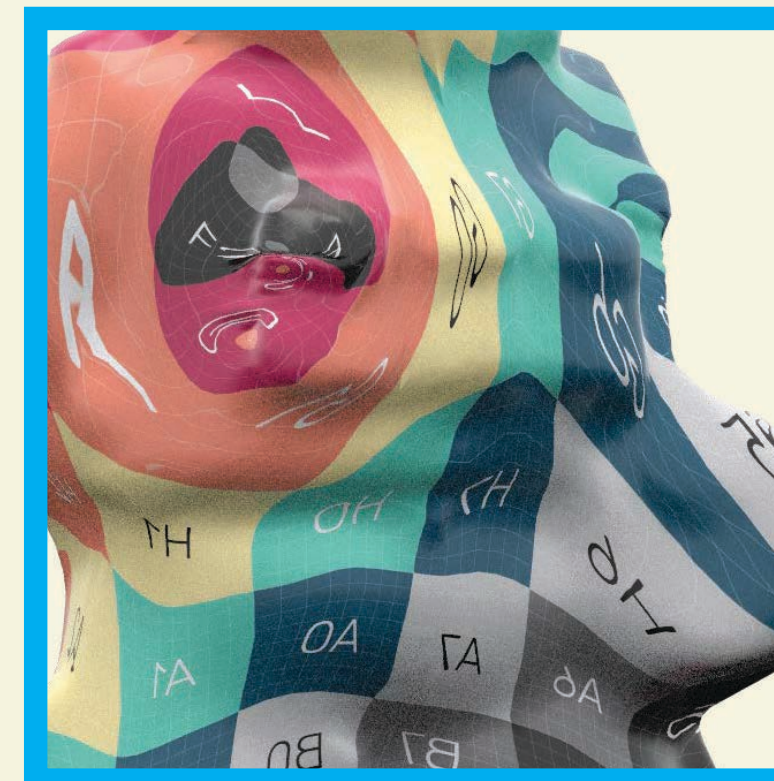


distortion

(symmetric Dirichlet)

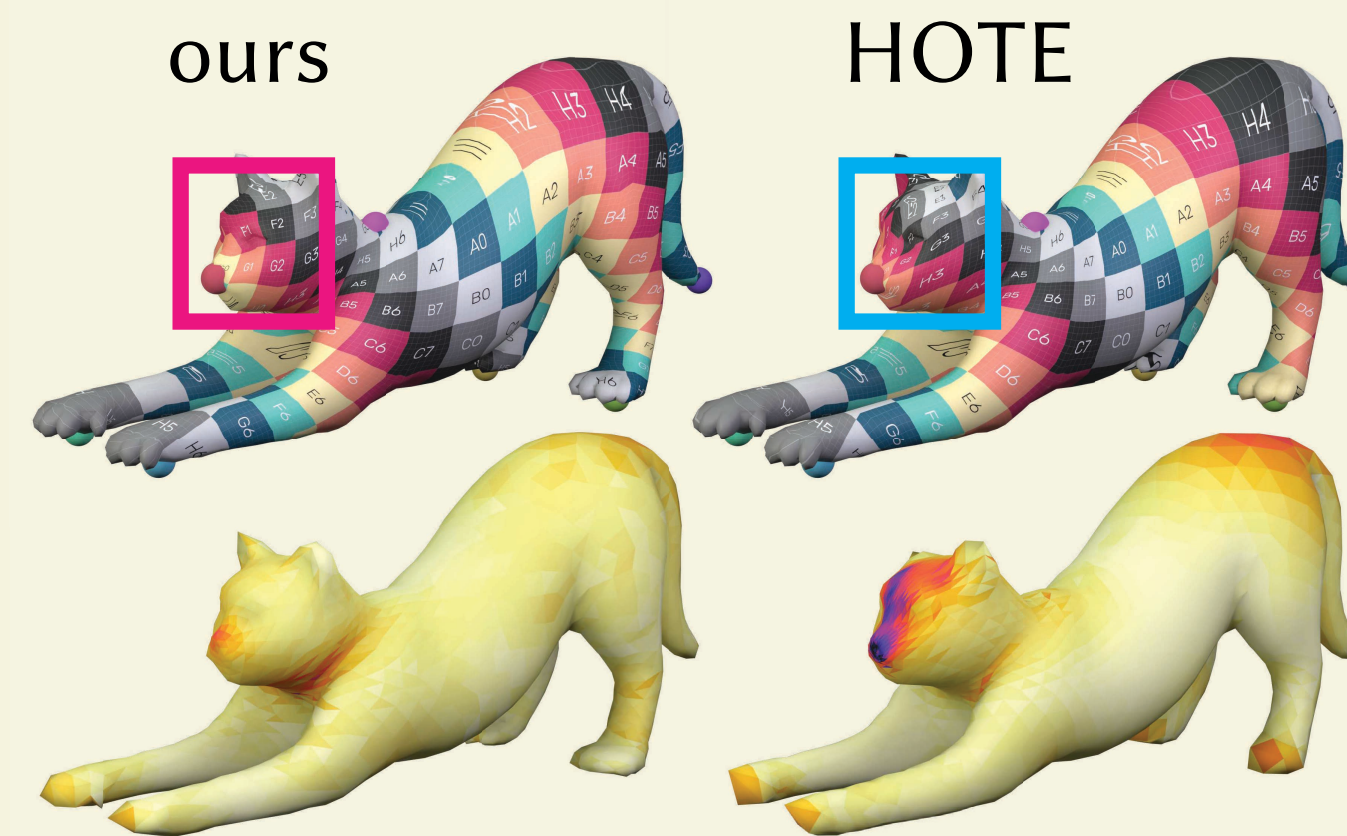


map
[Schmidt et al. 2023]



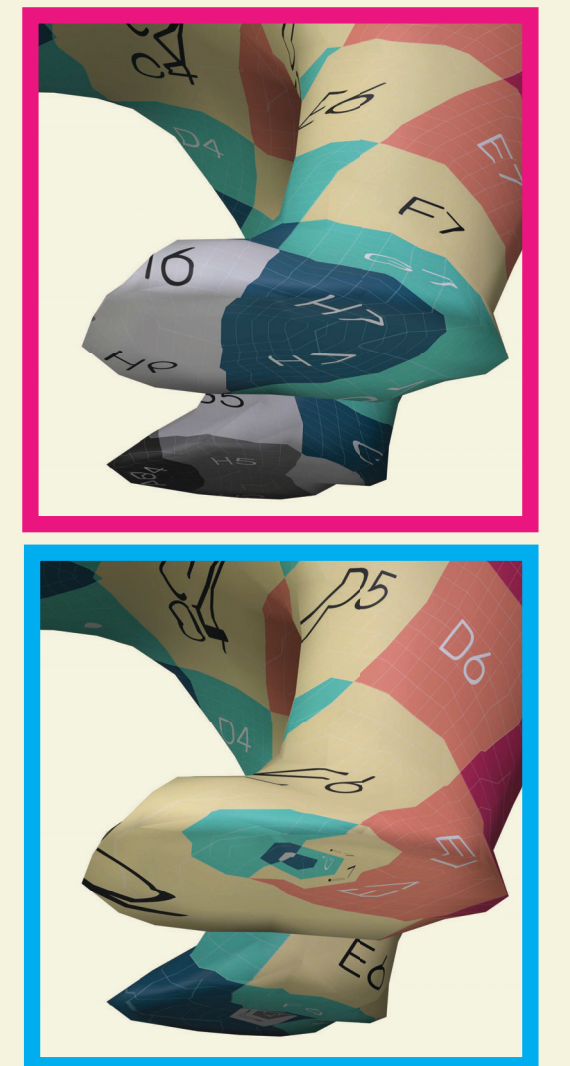
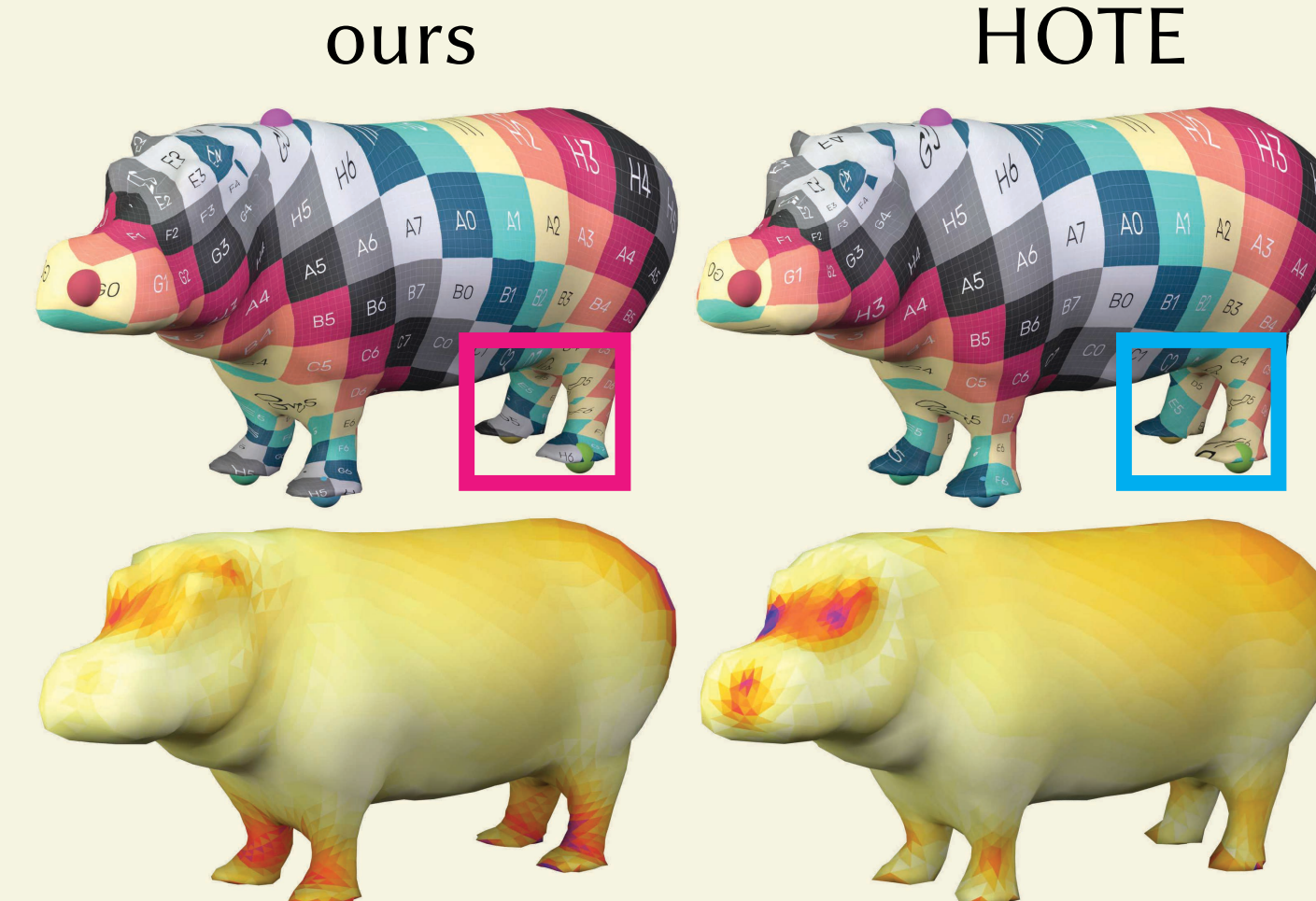
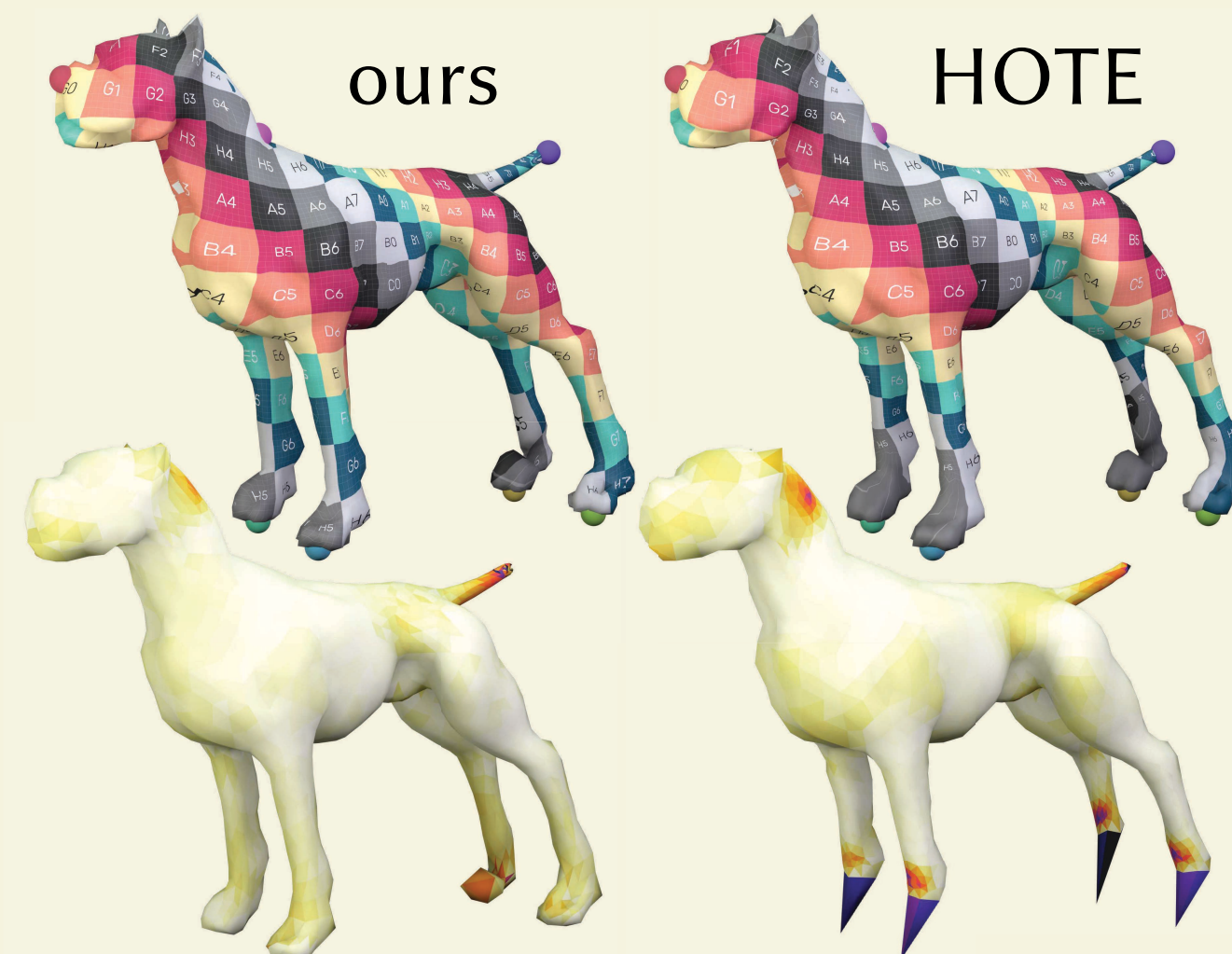
vs Hyperbolic Orbifold Tutte Embedding

[Aigerman and Lipman 2016]



Symmetric Dirichlet Energy

	ours	HOTE
cat	2.7	13.5
hippo	8.9	25582.5
dog	3.4	14.8



vs Adaptive Triangulations [Schmidt et al. 2023]

human femur

Source



Symmetric Dirichlet Energy

	ours	AT
bear	1.94	2.16
deer	2.02	2.22
pig	2.41	2.55

bear

Ours



AT

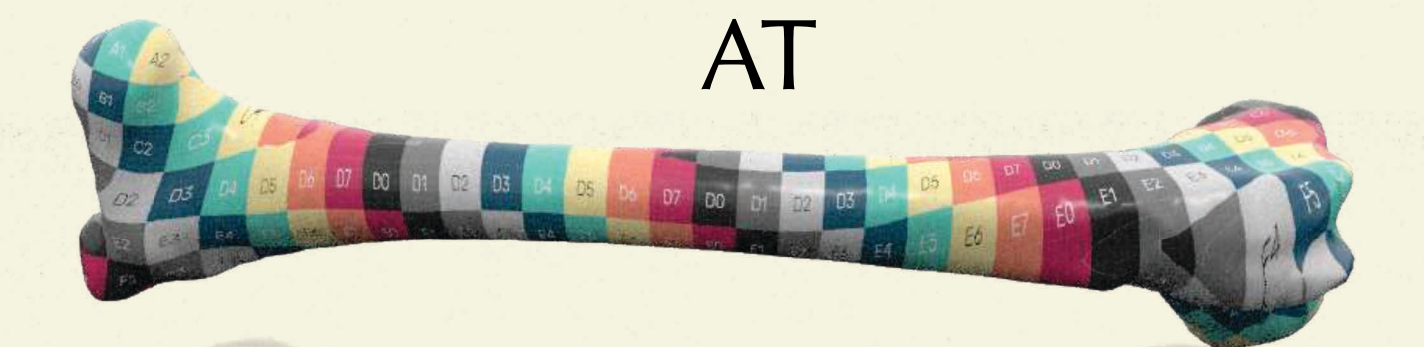


deer

Ours

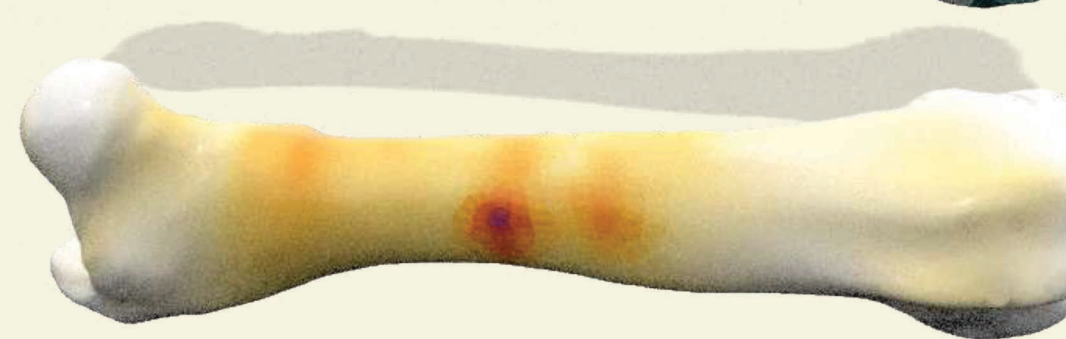


AT



pig

Ours

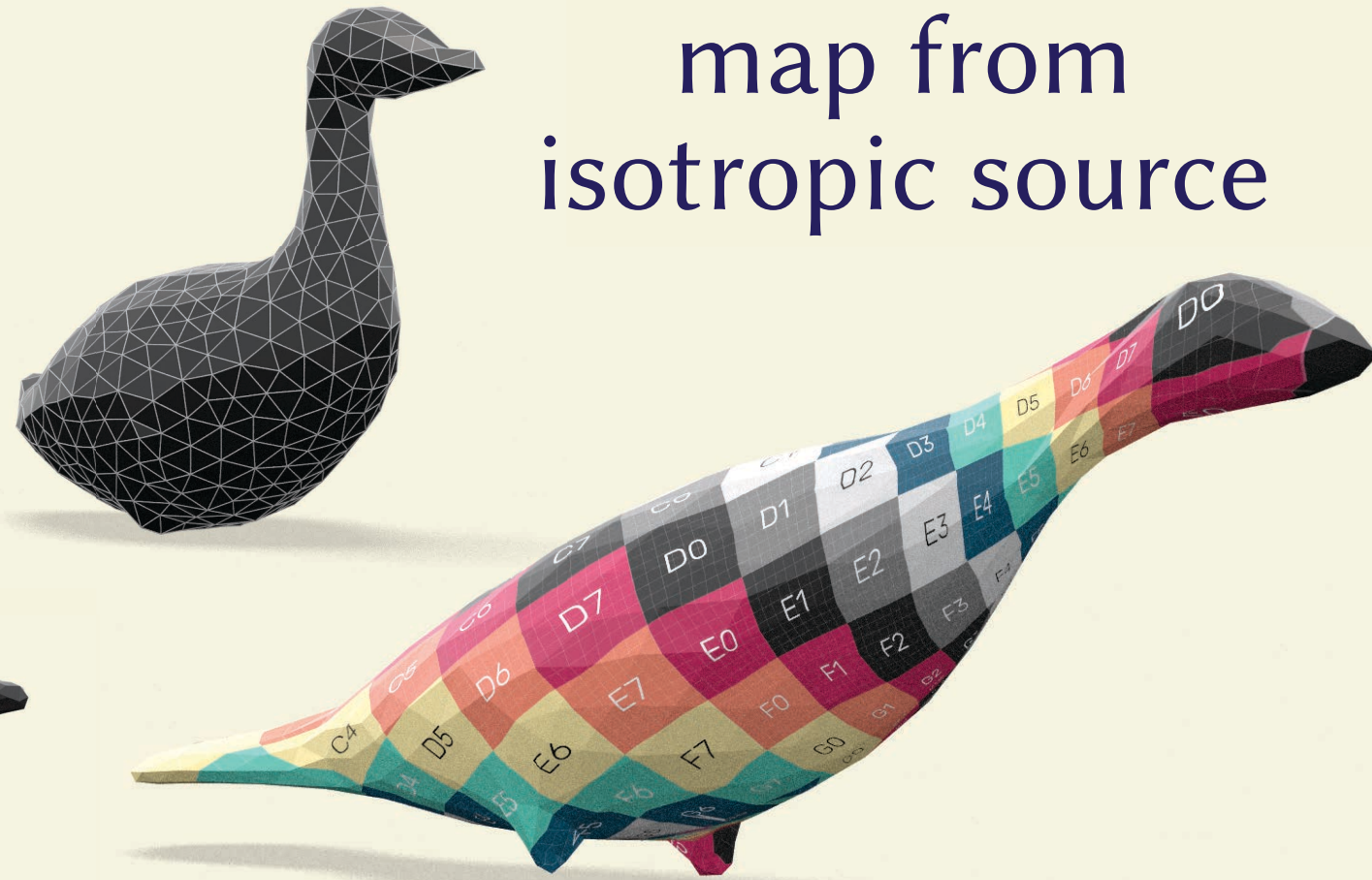


AT

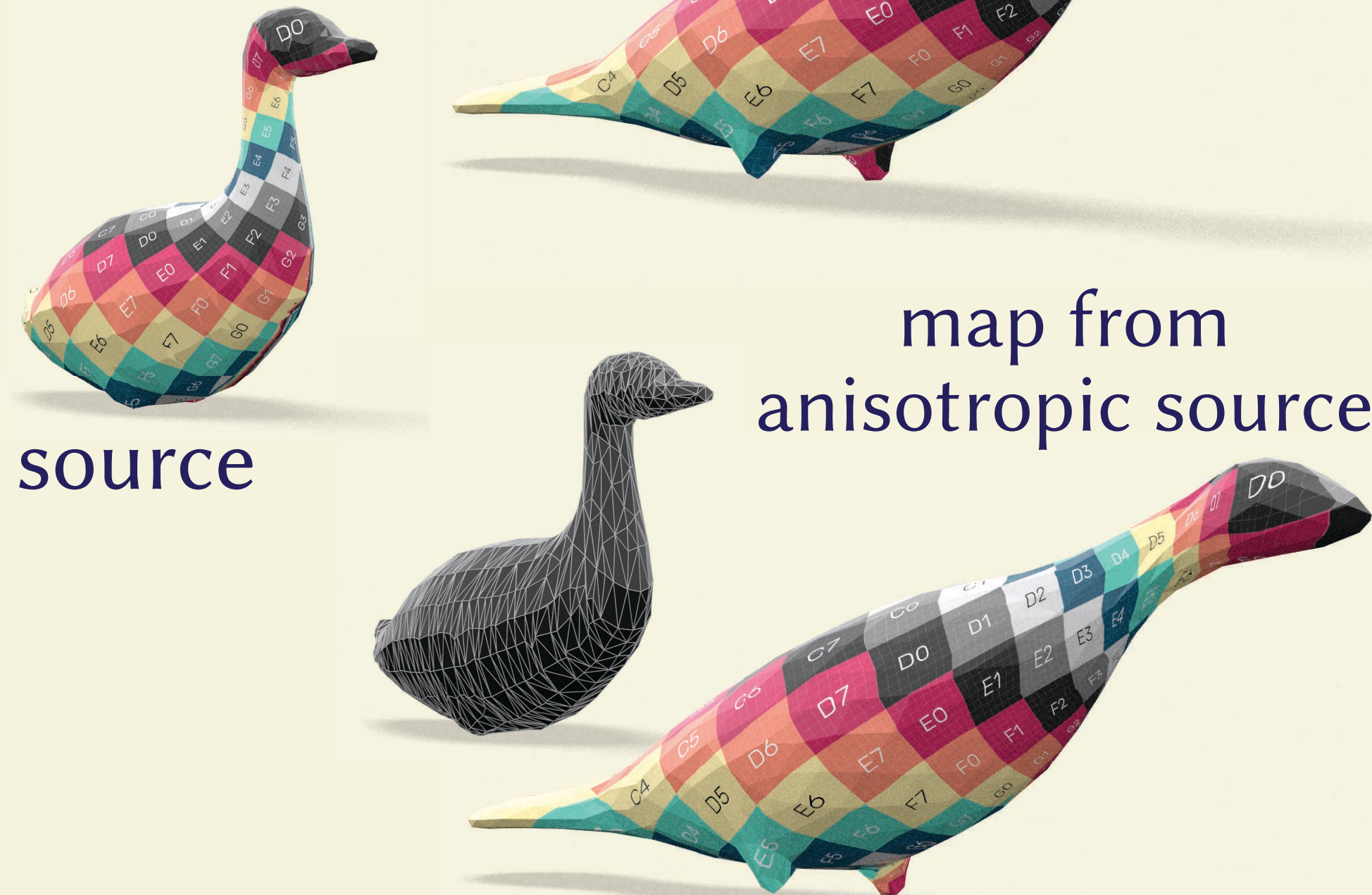


Robustness to Triangle Quality

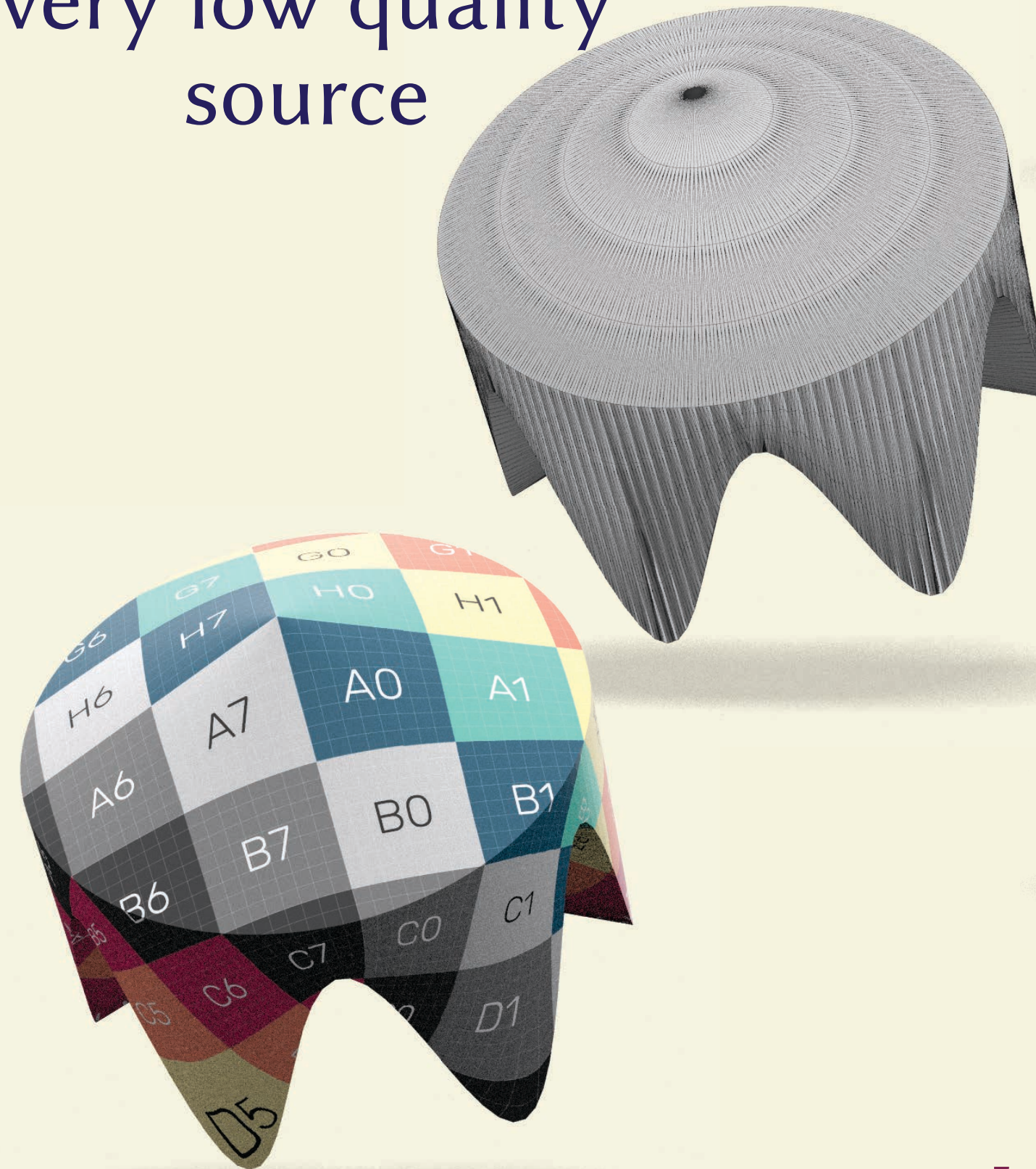
map from
isotropic source



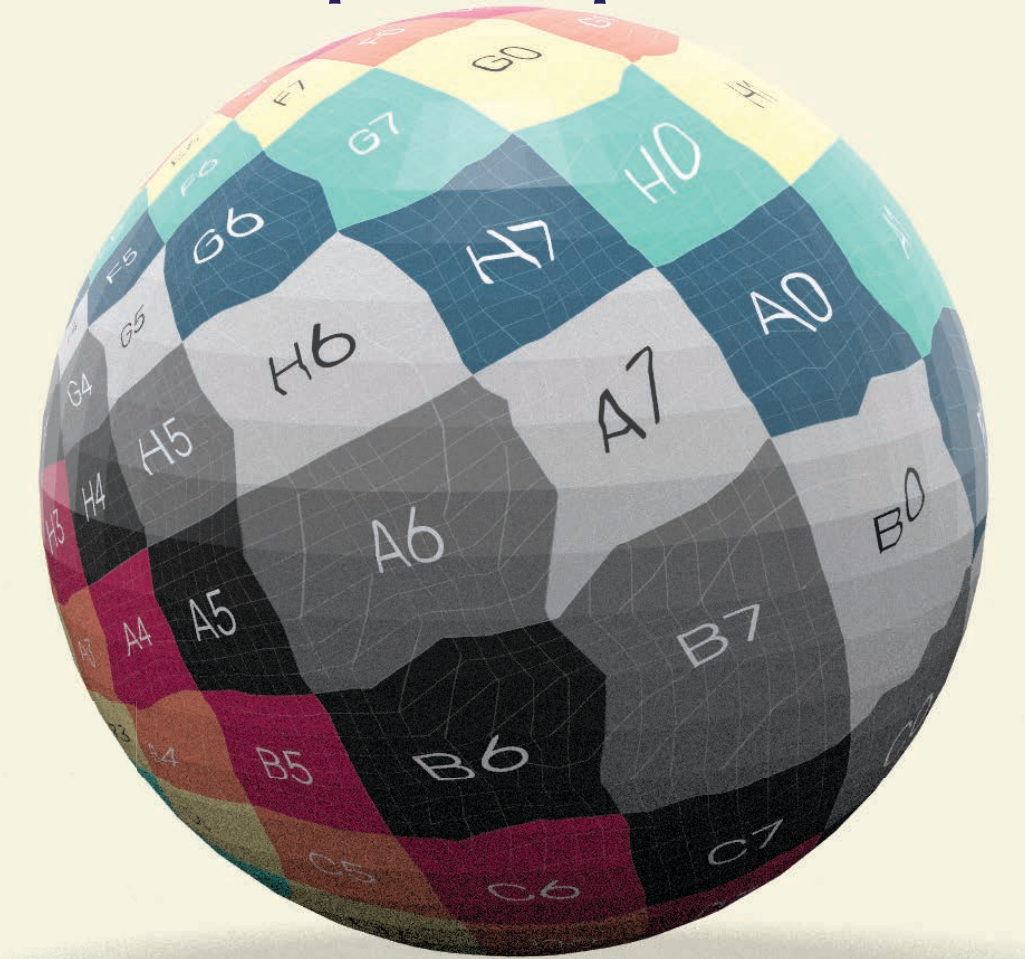
map from
anisotropic source



very low quality
source



map to sphere

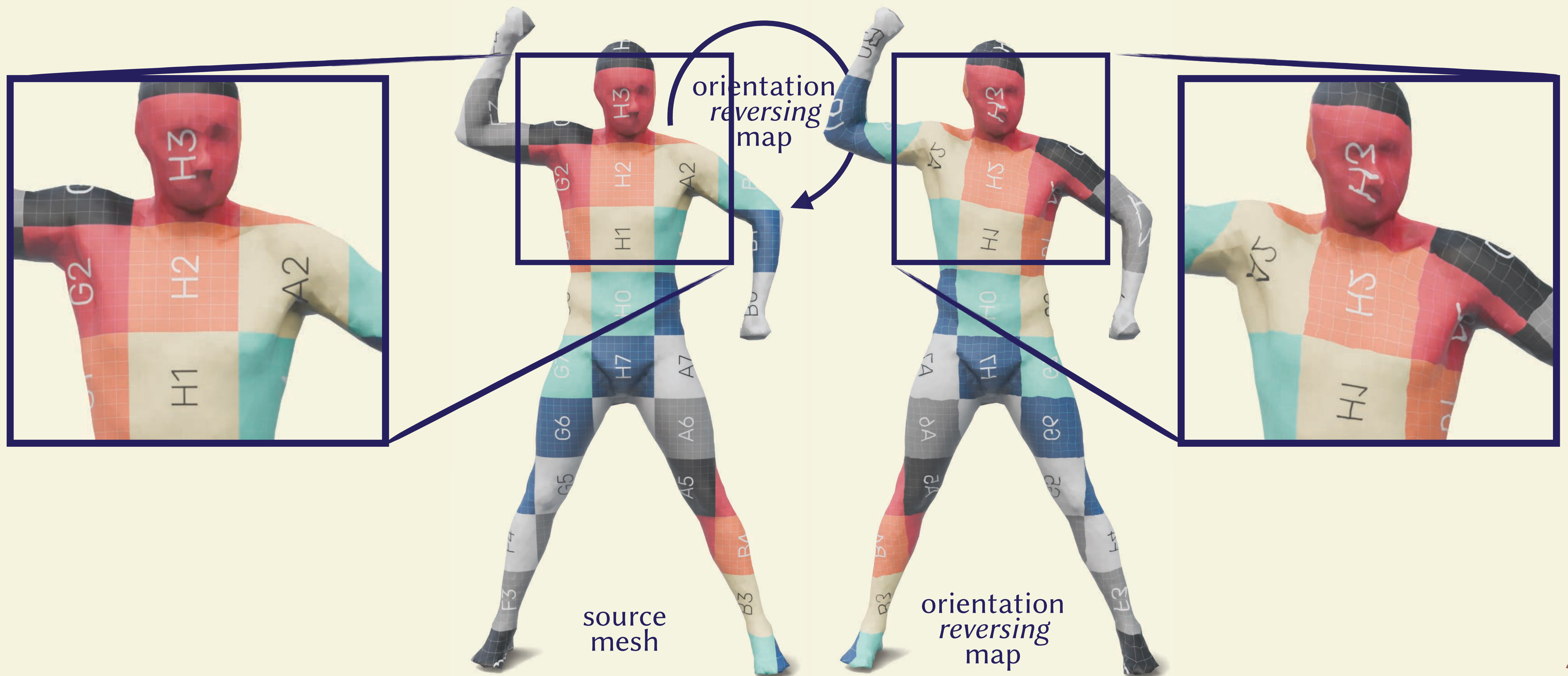


map to sphere using
intrinsic Delaunay
triangulation

[Bobenko and Springborn 2007;
Gillespie et al. 2021]

Orientation-Reversing Maps

construct line bundle with
first Chern class $\left[\frac{1}{2\pi}\Omega^\nabla\right]$
dual to anti-diagonal class
 $[\Delta] = [\text{pt}_A \times B - A \times \text{pt}_B]$



Covering Maps

construct line bundle with
first Chern class $\left[\frac{1}{2\pi}\Omega^\nabla\right]$
dual to double-covering class
 $[\Delta] = [\text{pt}_A \times B + 2A \times \text{pt}_B]$

A



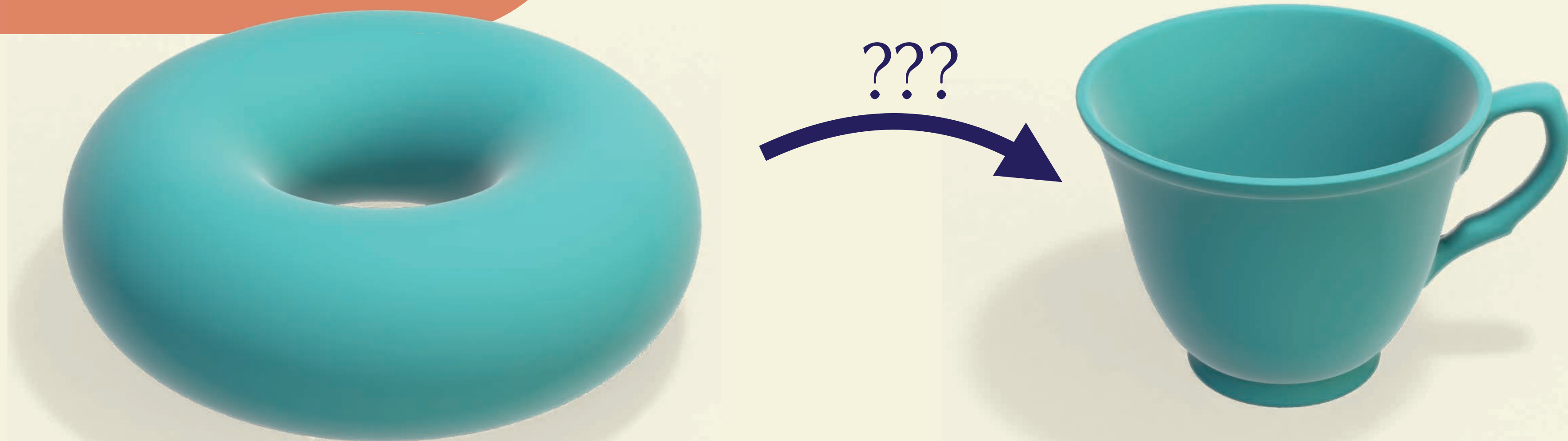
every point of A
appears twice on B

B



4 Open Questions

Genus $g > 0$



simple change: diagonal homology class has extra terms from H_1 generators

$g = 0$: associated varifold concentrates on a stationary varifold

$g > 0$: varifold may include *diffuse term* $\sim$ norm of a harmonic 1-form

Fix: optimize over both section *and* connection [Pigati & Stern 2021]

$$E_\varepsilon(u, \nabla) = \int_M |\nabla u|^2 + \frac{1}{4\varepsilon} (1 - |u|^2)^2 + \varepsilon^2 |\Omega^\nabla|^2 \quad (\text{Abelian Yang-Mills-Higgs energy})$$

Genus $g > 0$

Challenge: careful balancing between terms

Remark 1.2. We warn the reader, however, that while the qualitative analysis of the Allen–Cahn functionals does not depend on the precise choice of the double-well potential W , the analysis of the abelian Yang–Mills–Higgs functionals (1.1)–(1.2) seems to depend *quite strongly* on the choice $W(u) = \frac{1}{4}(1 - |u|^2)^2$. Indeed, already in two dimensions, replacing W with a potential $W_\lambda(u) := \frac{\lambda}{4}(1 - |u|^2)^2$ for some $\lambda \neq 1$ yields a dramatically different qualitative behavior, breaking the symmetry which leads to the first-order equations (1.4), and introducing interactions between disjoint components of the zero set (see, e.g., [21, Chapters I–III]). This should serve as one indication that the analysis of the abelian Higgs model is somewhat more delicate than that of related semilinear scalar equations, in spite of the strong parallels. [Pigati & Stern 2021]

and the optimization problem becomes much larger, and harder to solve

simple change: diagonal homology class has extra terms from H_1 generators

$g = 0$: associated varifold concentrates on a stationary varifold

$g > 0$: varifold may include *diffuse term* $\sim$ norm of a harmonic 1-form

Fix: optimize over both section *and* connection [Pigati & Stern 2021]

$$E_\varepsilon(u, \nabla) = \int_M |\nabla u|^2 + \frac{1}{4\varepsilon} (1 - |u|^2)^2 + \varepsilon^2 |\Omega^\nabla|^2 \quad (\text{Abelian Yang-Mills-Higgs energy})$$

What can we prove?

so far, results are mostly empirical

Is a minimal surface in the diagonal homology class guaranteed to be the graph of a function? A bijection?

Is a Ginzburg-Landau critical point at finite ε guaranteed to produce the graph of a function?

a priori, just a relation

Can we compute the global minimizer of area within the diagonal homology class?

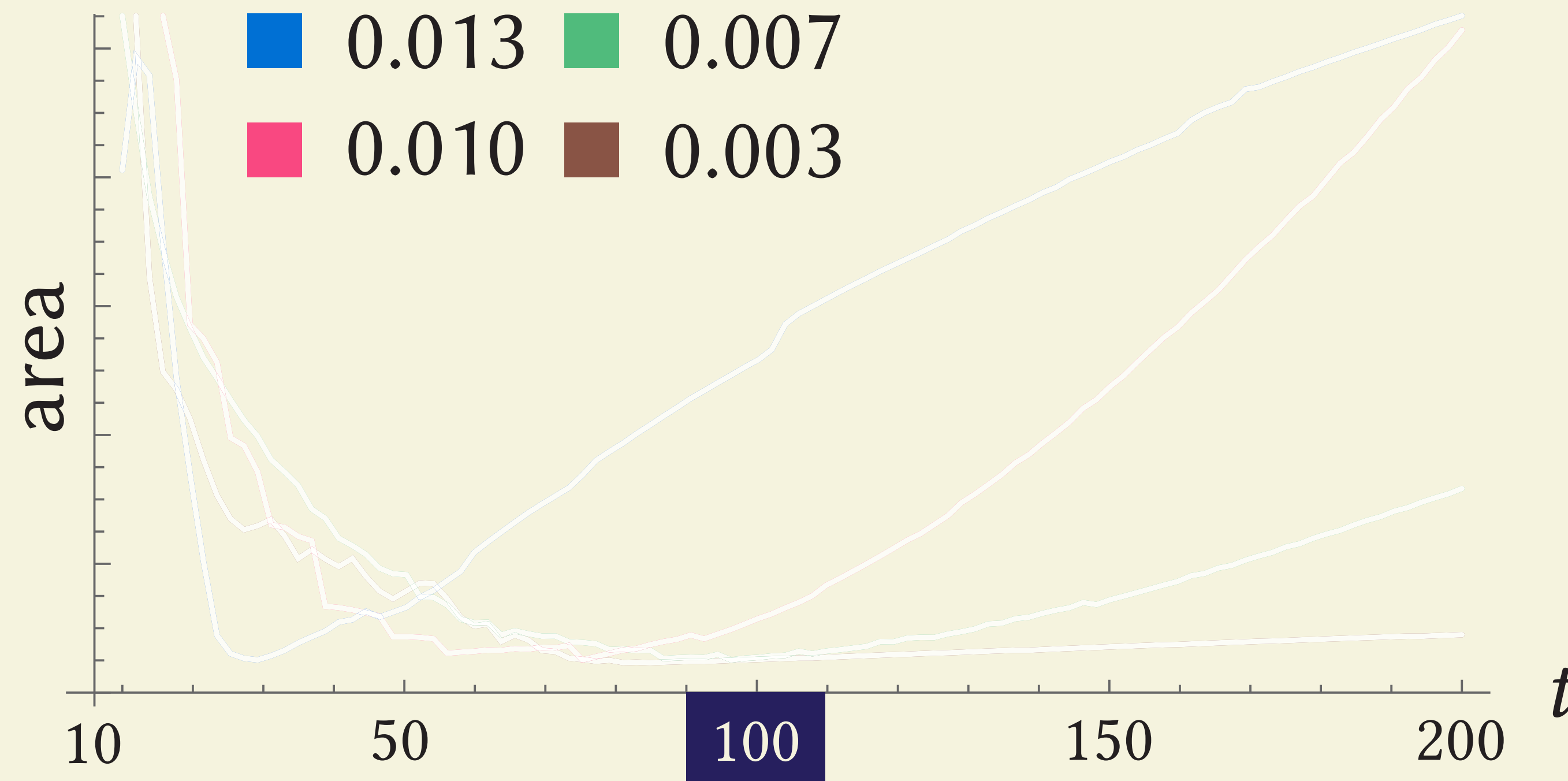
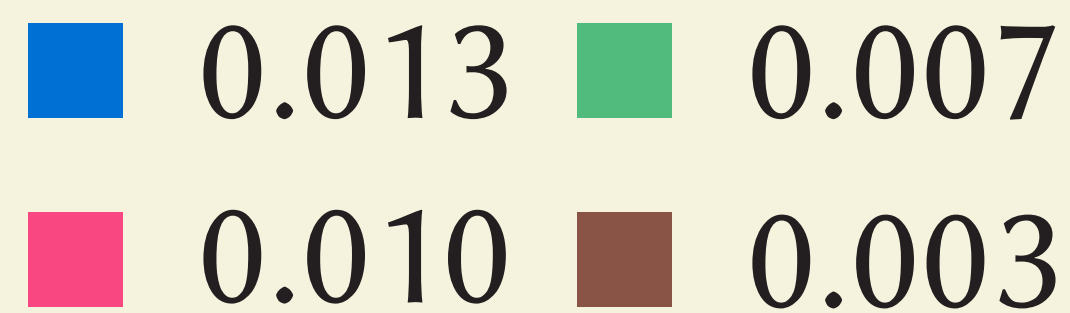
Convex *current relaxation* not tight in codimension-2 [Young 1963; De Lellis 2014]

Choice of Ginzburg-Landau Parameter

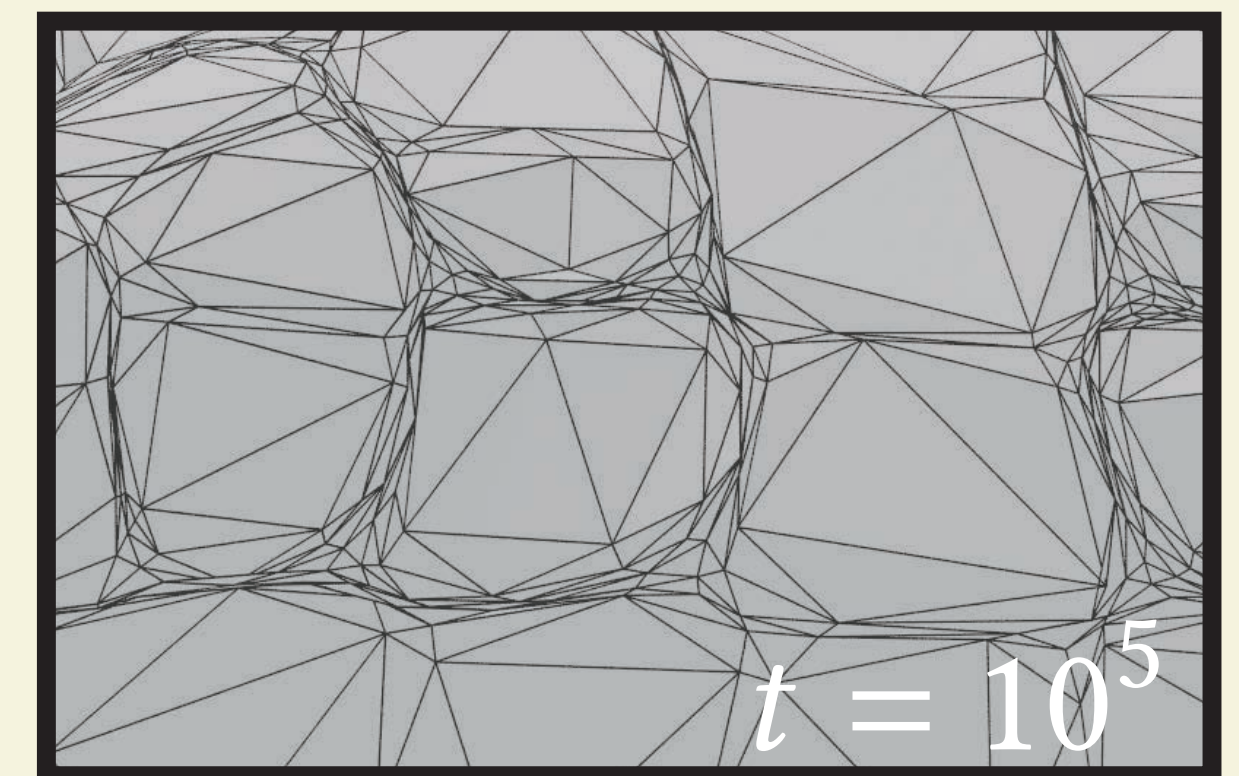
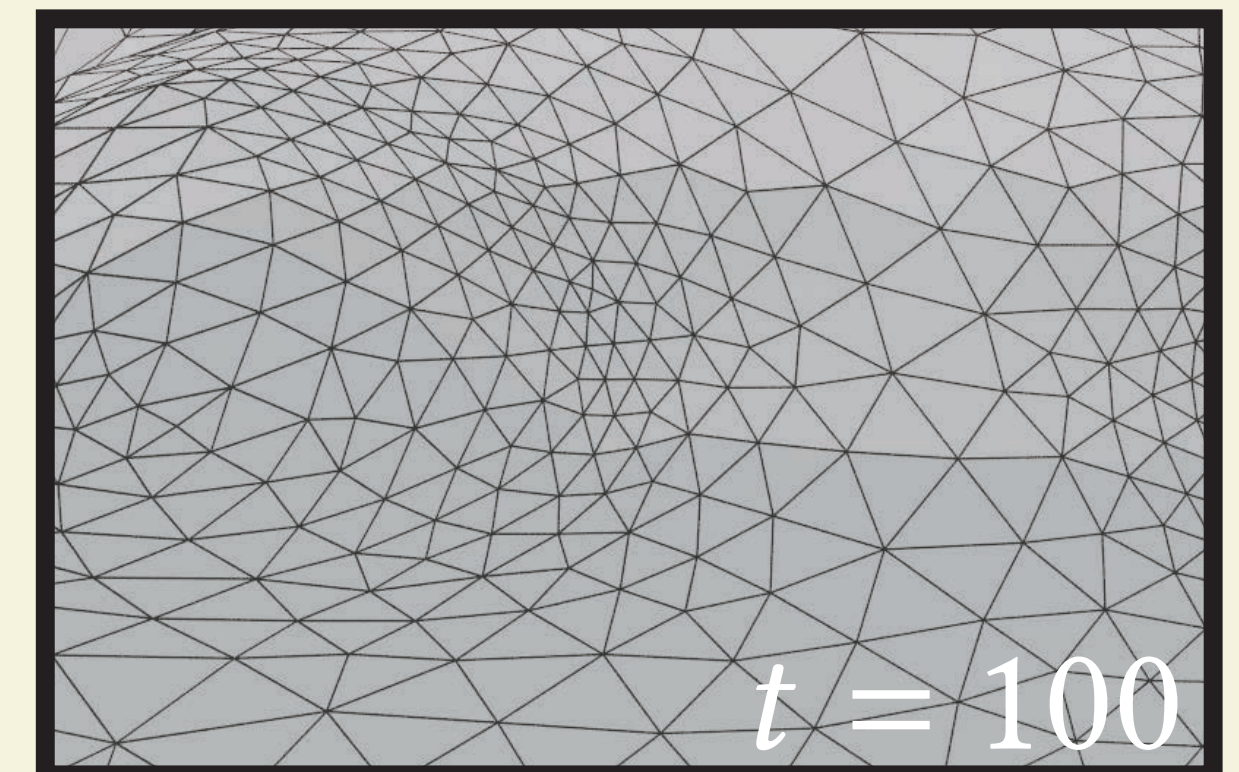
$$\mathcal{E}_\varepsilon^{\mathcal{GL}}(z) := \int_A \int_B \frac{1}{2} |\nabla z(p)|^2 + \frac{1}{4\varepsilon^2} \left(|z(p)|^2 - 1 \right)^2 dA dB$$

if $\frac{1}{\varepsilon^2} < \lambda_0$, the smallest eigenvalue of the connection Laplacian, the only critical point is 0
 $\implies$ take $\frac{1}{\varepsilon^2} = t\lambda_0$, for some constant $t > 1$

min edge lengths



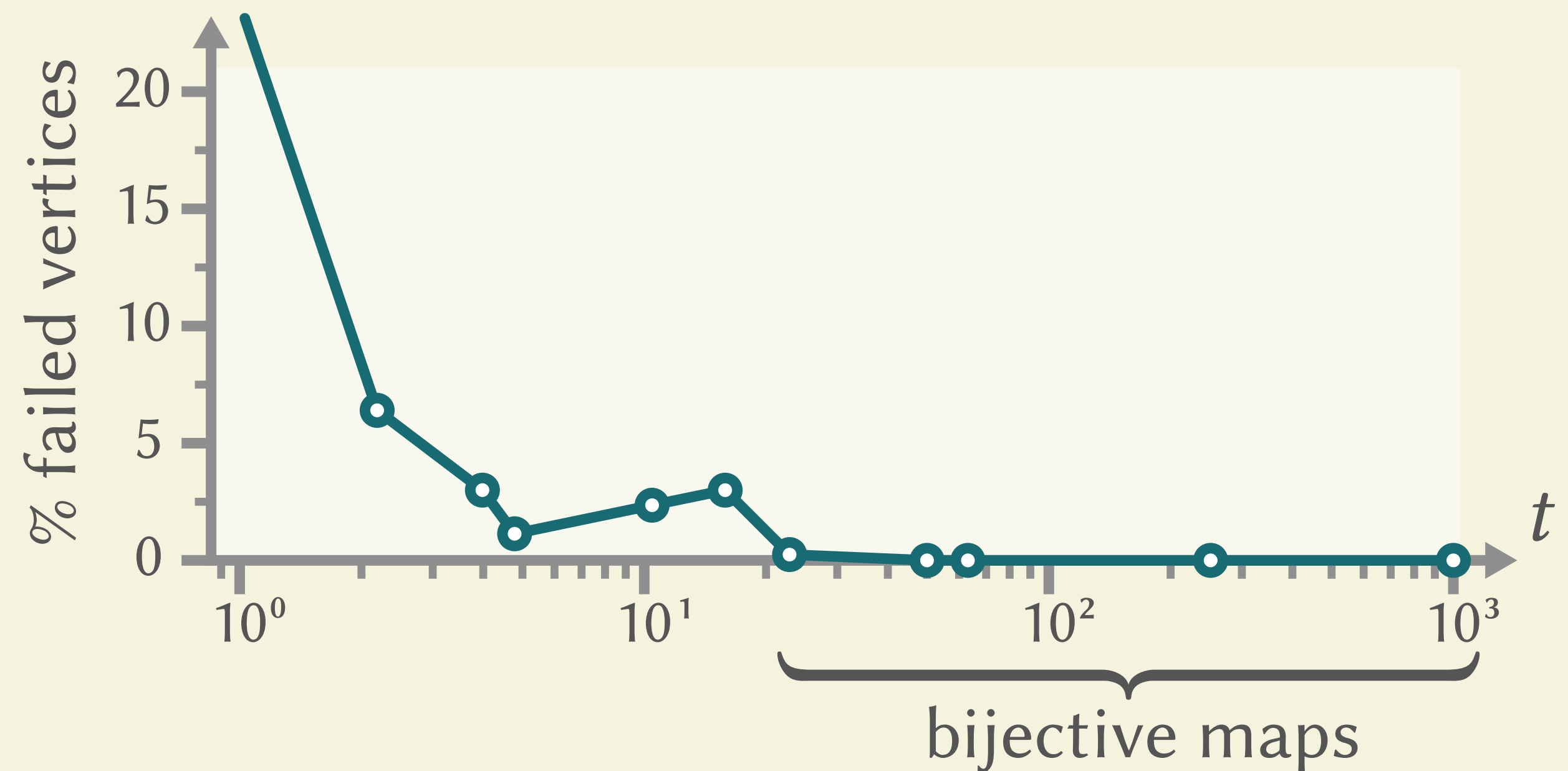
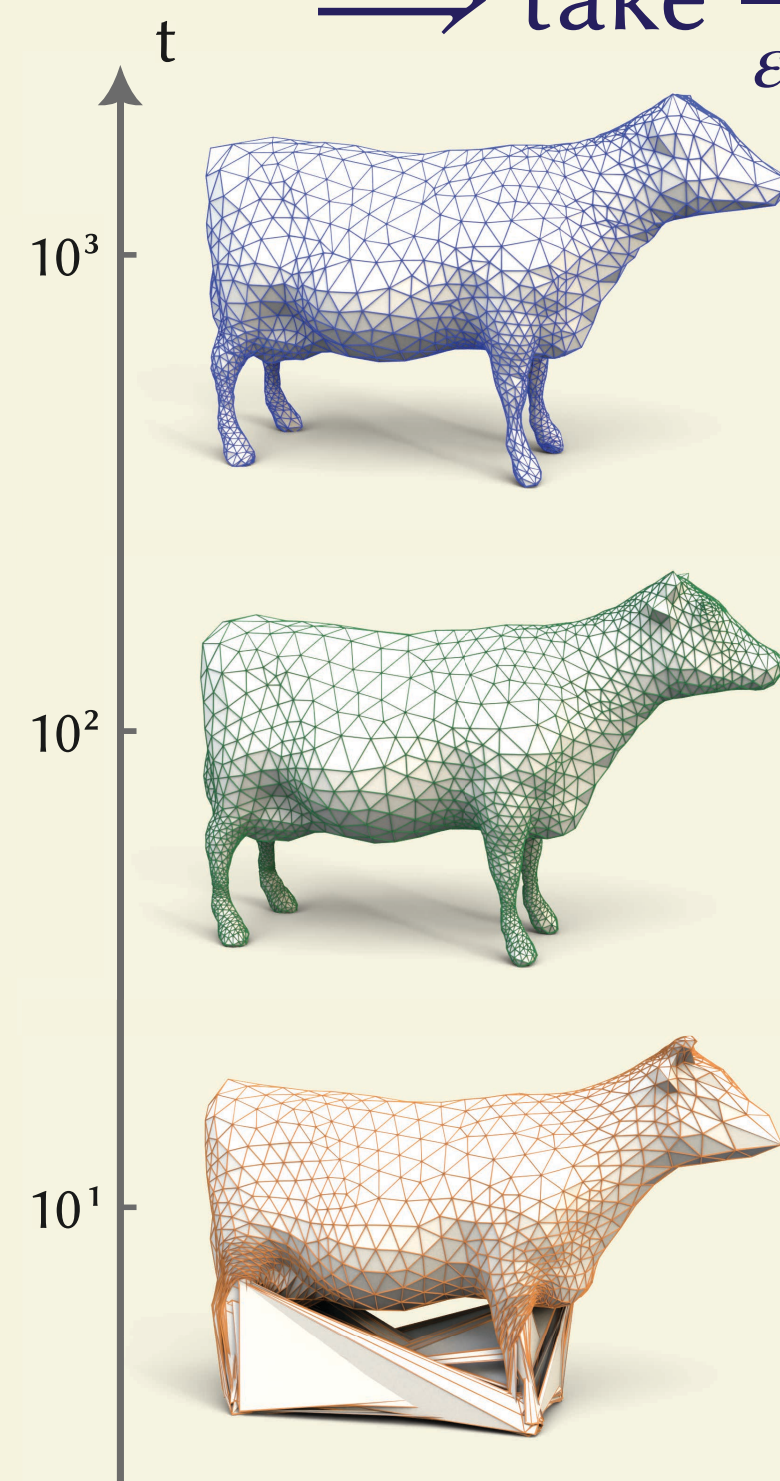
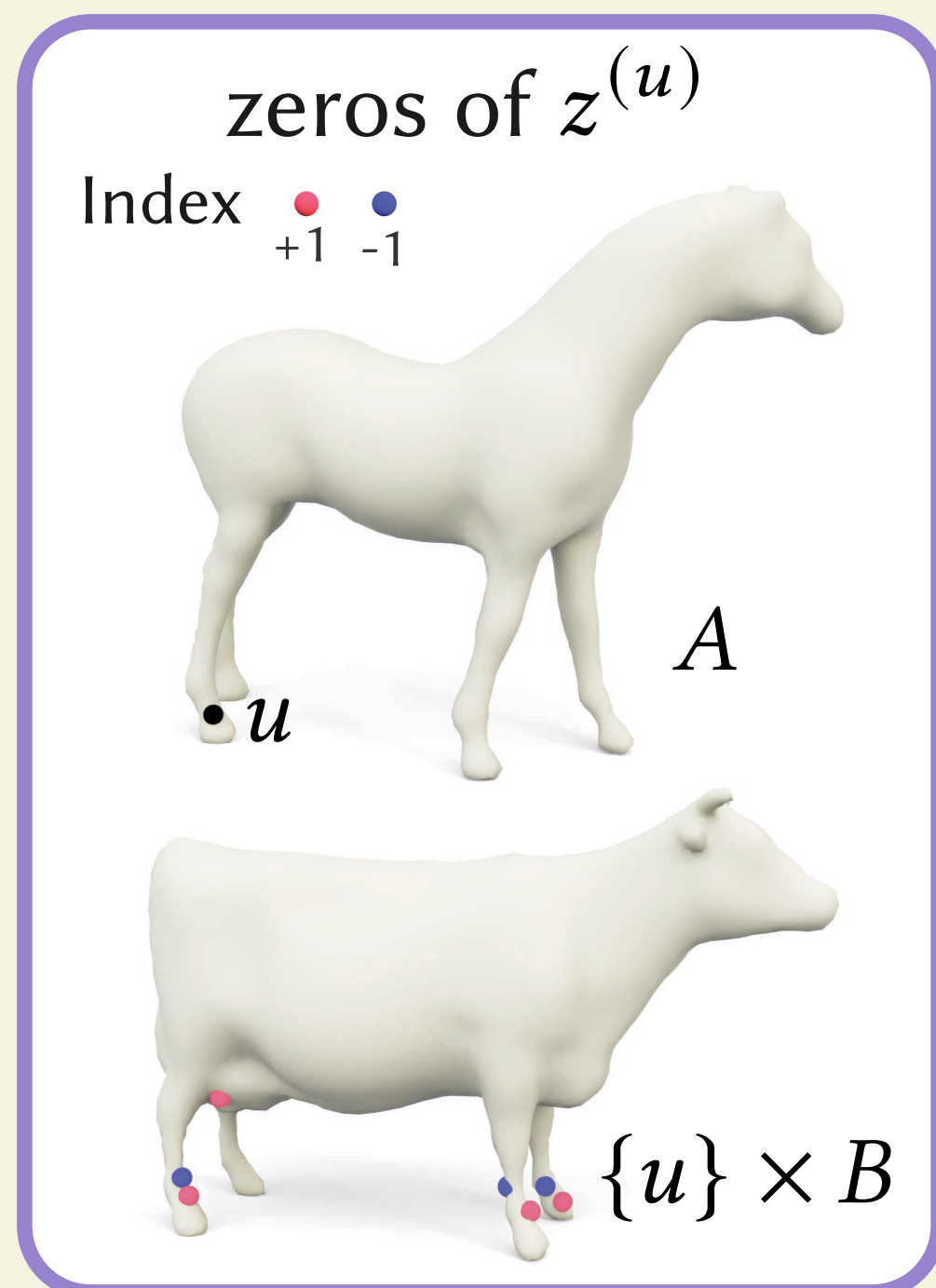
typical distortion



Picking the Ginzburg-Landau Parameter

$$\mathcal{E}_\varepsilon^{\mathcal{GL}}(z) := \int_A \int_B \frac{1}{2} |\nabla z(p)|^2 + \frac{1}{4\varepsilon^2} \left(|z(p)|^2 - 1 \right)^2 dA dB$$

if $\frac{1}{\varepsilon^2} < \lambda_0$, the smallest eigenvalue of the connection Laplacian, the only critical point is 0
 $\implies$ take $\frac{1}{\varepsilon^2} = t\lambda_0$, for some constant $t > 1$



Other surface energies

Codimension-1

minimal surfaces — Allen-Cahn energy [Modica and Mortola 1977]

Willmore surfaces [de Giorgi 1991]

arbitrary quadratic polynomials of principal curvatures [Song 2021]

Other surface energies

Codimension-1

minimal surfaces — Allen-Cahn energy [Modica and Mortola 1977]

Willmore surfaces [de Giorgi 1991]

arbitrary quadratic polynomials of principal curvatures [Song 2021]

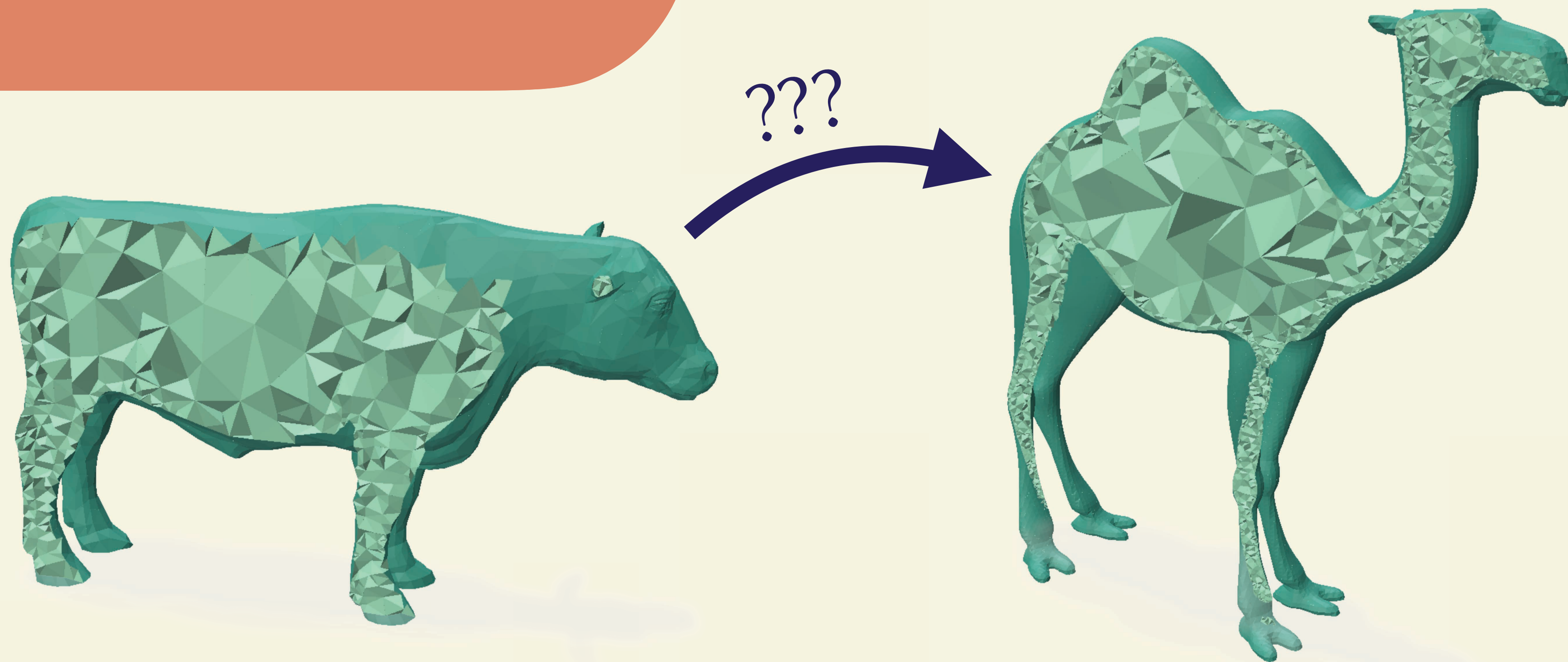
Codimension-2

minimal surfaces — Ginzburg-Landau / Yang-Mills-Higgs energies

Willmore surfaces (maybe the same?)

Other surface energies?

Volumetric Maps



Replace unit complex numbers by $SU(2)$ [Parise, Pigati & Stern 2025]

everything gets much harder

Thanks for listening

✉ mark.gillespie81@gmail.com

🌐 markjgillespie.com

